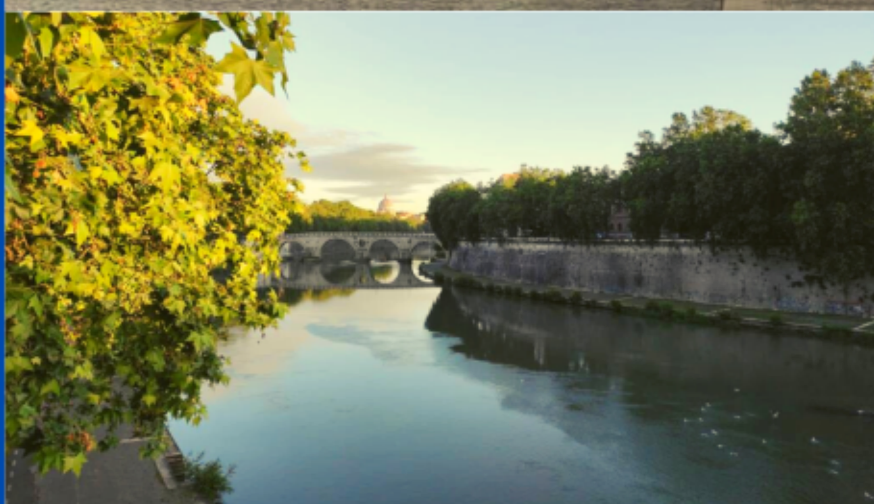
INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

ROMA • ONLINE
JULY 1 - 7, 2021

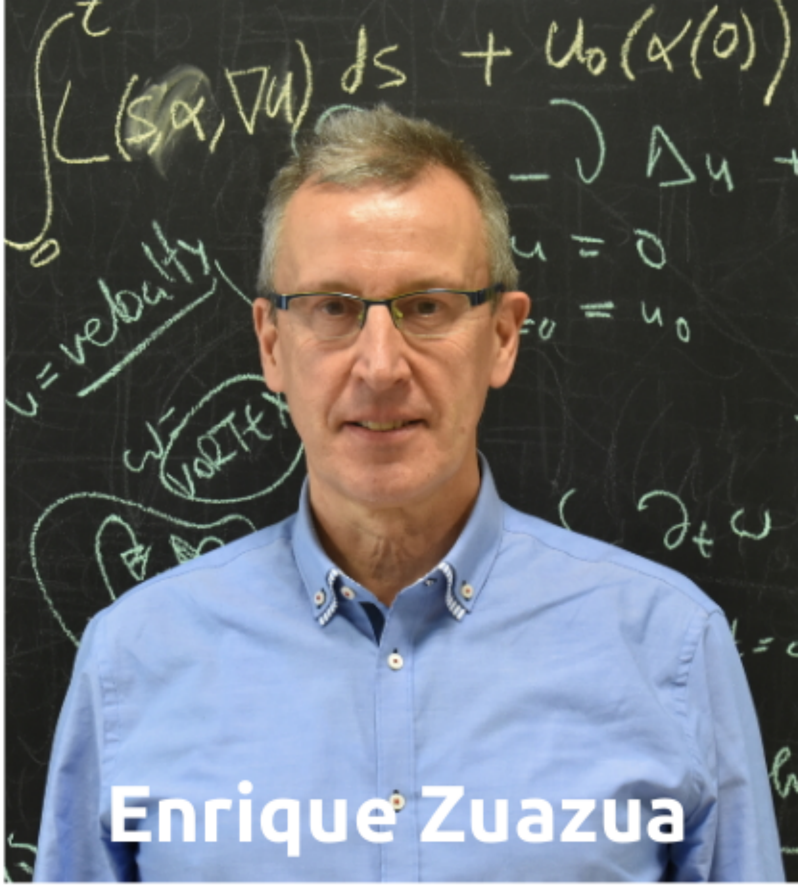
Book of Abstracts



INdAM
Istituto Nazionale di Alta Matematica

Contents


INdAM 2021
ANALYSIS AND
NUMERICS OF
DESIGN, CONTROL
AND INVERSE
PROBLEM



Scientific Committee's Note

The INdAM Workshop “**Analysis and Numerics of Design, Control and Inverse Problems**” selected by INdAM – Istituto Nazionale di Alta Matematica ‘F. Severi’ (Roma, Italy) will cover several topics in Applied Mathematics, including Control of PDEs, Shape Optimization, Inverse Problems and Numerical Analysis. It will be held in mixed mode: Roma and Online (Zoom and YouTube) from July 1 to July 7, 2021:
—Thursday 1 July – Saturday (morning) 3 July 2021 in mixed mode: both online and in presence (at most 30 participants in presence), the conference room will be located at the Globus Hotel in Roma
—Monday 5 July – Wednesday 7 July 2021, only online.

In many applications of mathematics one is confronted with the problem of steering a given system to a prescribed configuration, and often to do so in an optimal way. Sometimes this requires the optimal design of the shape of a body. All these issues can be addressed with a common body of techniques, ranging from functional analysis to partial differential equations, always in strict contact with numerical analysis for necessary implementation. Surprisingly enough, some of these techniques can be used to recover information on parameters from measurements of the solution. These are so-called inverse problems which play an important role in climate science and bio mathematics.

The aim of this conference is to bring together many experts in the above fields to explore the possibility of collaboration. We particularly expect the participation of young researchers, for whom the conference is dedicated.

See you in Roma / Online!

Giuseppe Floridia
Università Mediterranea di Reggio Calabria & INdAM Unit of the University of Catania

Enrique Zuazua
FAU Erlangen-Nürnberg – Alexander von Humboldt Prof.
Deusto Foundation-Bilbao | UAM-Madrid

INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

Invited Speakers

ROMA • ONLINE | JULY 1 - 7, 2021

BOOK OF ABSTRACTS

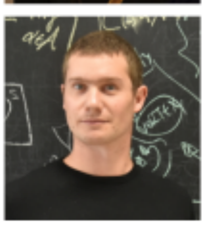
Invited Speakers

Harbir Antil
George Mason University, US

Karine Beauchard
ENS Rennes, France

Nicola Bellomo
Polytechnic of Torino, Italy

Roberta Bianchini
CNR | National Research Council, Italy



Umberto Biccari
University of Deusto | DeustoCCM, Bilbao - Spain

Lucio Boccardo
Sapienza University of Roma, Italy

Invited Speakers

Lorena Bociu
NC State University, US

Francesca Bucci
University of Firenze, Italy

Martin Burger
FAU Erlangen-Nürnberg, Germany

Giuseppe Buttazzo
University of Pisa, Italy

Fabio Camilli
Sapienza University of Roma, Italy

Piermarco Cannarsa
University of Roma “Tor Vergata”, Italy

Invited Speakers

Marco Caponigro
University of Roma "Tor Vergata", Italy

Sandra Carillo
Sapienza University of Roma, Italy

Pierluigi Colli
University of Pavia, Italy

Emmanuelle Crepeau
University of Grenoble, France

Sylvain Ervedoza
University of Bordeaux, France

Maurizio Falcone
Sapienza University of Roma, Italy

Invited Speakers

Genni Fragnelli
University of Bari, Italy

Martin Gugat
FAU Erlangen-Nürnberg, Germany

Roberto Guglielmi
University of Waterloo, Canada

Michael Hintermüller
Humboldt Berlin University, Germany

Oleg Y. Imanuvilov
Colorado State University, US

Yavar Kian
Aix-Marseille University, France

Invited Speakers

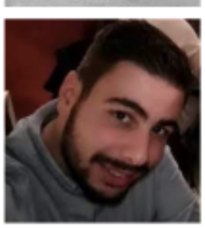
Dongnam Ko
Catholic University of Korea, Korea

Martin Lazar
University of Dubrovnik, Croatia

Günter Leugering
FAU Erlangen-Nürnberg, Germany

Paola Loreti
Sapienza University of Roma, Italy

Cristian Mendico
GSSI L'Aquila, Italy



Alessandro Paolucci
University of L'Aquila, Italy

Invited Speakers

Cristina Pignotti
University of L'Aquila, Italy

Alessio Porretta
University of Roma "Tor Vergata", Italy

Camille Pouchol
University of Paris, France

Alfio Quarteroni
Mox, Politecnico of Milano, Italy

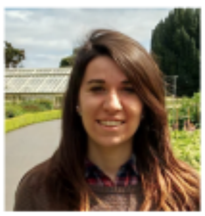
Biagio Ricceri
University of Catania, Italy

Elisabetta Rocca
University of Pavia, Italy

Invited Speakers

Gianluigi Rozza
SISSA, Trieste, Italy

Domènec Ruiz-Balet
UAM | DeustoCCM, Bilbao – Spain



Teresa Scarinci
University of L'Aquila, Italy

Eric Soccorsi
Aix-Marseille University

Hiroshi Takase
The University of Tokyo, Japan

Emmanuel Trélat
Sorbonne University, Paris – France

Invited Speakers

Cristina Trombetti
University of Naples Federico II, Italy

Marius Tucsnak
University of Bordeaux, France

Cristina Urbani
University of Roma “Tor Vergata”, Italy

Enrico Valdinoci
The University of Western Australia

Judith Vancostenoble
Paul Sabatier University, Toulouse III, France

Masahiro Yamamoto
The University of Tokyo, Japan

Christophe Zhang
FAU Erlangen-Nürnberg, Germany

INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

Program

ROMA • ONLINE | JULY 1 - 7, 2021

BOOK OF ABSTRACTS

Program

Thursday July 1, 2021 [Roma & online]
09:45 – 10:00 [Opening]
10:00 – 10:35 Piermarco Cannarsa – University of Roma "Tor Vergata": "Bilinear control for evolution equations"
10:40 – 11:15 Masahiro Yamamoto – The University of Tokyo: "Carleman estimates and inverse problems for transport equation"
11:20 – 11:55 Coffee break
11:55 – 12:30 Sylvain Ervedoza – University of Bordeaux: "Observability of waves in an annulus for various boundary conditions"
12:35 – 13:10 Judith Vancostenoble – Paul Sabatier University: "Lipschitz stability for the growth rate coefficients in a nonlinear Fisher-KPP equation"
13:15 – 15:00 Lunch
15:00 – 15:35 Giuseppe Buttazzo – University of Pisa: "Upper and lower bounds for some shape functionals"
15:40 – 16:15 Elisabetta Rocca – University of Pavia: "Optimal control and log-time behavior of diffuse interface models of tumor growth"
16:20 – 16:55 Coffee break
16:55 – 17:20 Roberta Bianchini – CNR, Italy: "Some recent results on 2D stably stratified fluids"
17:25 – 18:00 Fabio Camilli – Sapienza University of Roma: "A regularity theory for viscous Hamilton-Jacobi equations with Caputo time-fractional derivative"

Friday July 2, 2021 [Roma & online]
09:00 – 09:35 Paola Loreti – Sapienza University of Roma: "Memory Effects and PDEs in Glass Relaxation"
09:40 – 10:05 Cristina Urbani – University of Roma "Tor Vergata": "A constructive algorithm for building mixing coupling potentials. Application to bilinear control"*
10:10 – 10:45 Biagio Ricceri – University of Catania: "Recent results on variational inequalities"
10:50 – 11:25 Coffee break
11:25 – 12:00 Cristina Pignotti – University of L'Aquila: "Consensus of opinion formation models with time delay"
12:05 – 12:30 Alessandro Paolucci – University of L'Aquila: "On the control of the Hegselmann-Krause model with leadership and time delay"*
12:35 – 14:30 Lunch
14:30 – 15:05 Martin Lazar – University of Dubrovnik: "Optimal control of parabolic equations – a spectral calculus based approach"
15:10 – 15:35 Umberto Biccari – DeustoCCM | University of Deusto: "Multilevel Selective Harmonic Modulation via Optimal Control"*
15:40 – 16:15 Francesca Bucci – University of Firenze: "On an optimal evolution in linear-quadratic control"
16:20 – 16:55 Coffee break
16:55 – 17:20 Cristian Mendico – GSSI L'Aquila, Italy: "Ergodic behavior of solutions to Hamilton-Jacobi equations for sub-Riemannian control systems"
17:25 – 18:00 Sandra Carillo – Sapienza University of Roma: "The integro-differential model of viscoelastic body: thermodynamically admissible kernels"
20:00 – 22:00 Social dinner

Program

Saturday July 3, 2021 [Roma & online]
09:00 – 09:35 Alfio Quarteroni – Mox, Politecnico of Milano: "The Integrated Heart"
09:40 – 10:15 Gianluigi Rozza – SISSA, Trieste: "An Overview of Reduced Order Methods for Optimal Flow Control Problems"
10:20 – 10:55 Karine Beauchard – ENS Rennes: "On expansions for nonlinear systems, error estimates and convergence issues"
11:00 – 11:35 Coffee break
11:35 – 12:10 Marco Caponigro – University of Roma "Tor Vergata": "Exact controllability in projections of the bilinear Schrödinger equation"
12:15 – 12:50 Lucio Boccardo – Sapienza University of Roma: "Regularizing effect of the interplay between coefficients in Dirichlet problems with convection or drift terms"
12:55 – 13:00 [Closing part in presence at Roma]
13:00 – 15:00 Lunch
15:00 – 18:30 Social visits

Monday July 5, 2021 [online]
10:00 – 10:35 Emmanuel Trélat – Sorbonne University: "Robustness of various controllability properties under sampling"
10:40 – 11:05 Camille Pouchol – University of Paris: "Unwanted sparsity for linear inverse problems with non-negativity constraints"*
11:10 – 11:40 Virtual Coffee break
11:40 – 12:15 Martin Gugat – FAU Erlangen-Nürnberg: "Some remarks on the turnpike property"
12:20 – 12:55 Michael Hintermüller – Humboldt Berlin University: "Optimization with learning-informed differential equation constraints and its applications"
13:00 – 15:00 Virtual Lunch
15:00 – 15:35 Alessio Porretta – University of Roma "Tor Vergata": "Effects of gradient blow-up in viscous Hamilton-Jacobi equations"
15:40 – 16:15 Maurizio Falcone – Sapienza University of Roma: "Dynamic Programming on a tree for the approximation of finite horizon optimal control problems"
16:15 – 16:45 Virtual Coffee break
16:45 – 17:20 Lorena Bociu – NC State University: "Multiscale interface coupling between a poroelastic medium and a lumped hydraulic circuit"
17:25 – 18:00 Pierluigi Colli – University of Pavia: "The viscous Cahn-Hilliard system and the sliding mode control problem"

Program

Tuesday July 6, 2021 [online]
10:00 – 10:35 Cristina Trombetti – University of Naples Federico II: "An optimization problem in thermal insulation"
10:40 – 11:05 Dongnam Ko – The Catholic University of Korea: "Model predictive control with random batch methods for a guiding problem"*
11:10 – 11:40 Virtual Coffee break
11:40 – 12:15 Yavar Kian – Aix-Marseille University: "Recovery of nonlinear terms for reaction-diffusion equations"
12:20 – 12:55 Emmanuelle Crepeau – University of Grenoble: "Internal null controllability of the Hirota-Satsuma system"
13:00 – 15:00 Virtual Lunch
15:00 – 15:35 Genni Fragnelli – University of Bari: "Boundary Controllability for a degenerate wave equation in non divergence form with drift"
15:40 – 16:05 Roberto Guglielmi – University of Waterloo, Canada: "Sensitivity analysis of the value function for semilinear parabolic optimal control problems"*
16:10 – 16:35 Teresa Scarinci – University of L'Aquila, Italy: "Some optimal control problems under uncertainty and convergence analysis for stochastic proximal gradient methods"*
16:40 – 17:00 Virtual Coffee break
17:00 – 17:35 Harbir Antil – George Mason University: "Fractional Operators: Analysis and Optimal Control with Physical and Data Science Applications"
17:40 – 18:15 Nicola Bellomo – University of Granada | Polytechnic of Torino: "Modeling Virus Pandemics by Multiscale Methods"

Wednesday July 7, 2021 [online]
09:00 – 09:35 Enrico Valdinoci – The University of Western Australia: "The Lévy flight foraging hypothesis"
09:40 – 10:05 Christophe Zhang – FAU Erlangen-Nürnberg: "Shape control of the heat equation"*
10:10 – 10:45 Marius Tucsnak – University of Bordeaux, France: "Controllability of a class of infinite dimensional systems with age structure"
10:50 – 11:20 Virtual Coffee break
11:20 – 11:55 Martin Burger – FAU Erlangen-Nürnberg: "Kinetic Models and (Social) Networks"
12:00 – 12:35 Éric Soccorsi – Aix-Marseille University: "Do quantum currents uniquely determine the magnetic potential of an Iwatsuka Hamiltonian?"
12:40 – 15:00 Virtual Lunch

Program

…(Cont.) Wednesday July 7, 2021 [online]

12:40 – 15:00 Virtual Lunch
15:00 – 15:35 Oleg Y. Imanuvilov – Colorado State University: "Controllability to trajectories of a simplified fluid-structure interaction model"
15:40 – 16:05 Hiroshi Takase – University of Tokyo: "Inverse problems for first-order hyperbolic equations"*
16:10 – 16:40 Virtual Coffee break
16:40 – 17:05 Domènec Ruiz-Balet – UAM | DeustoCCM: "Control of reaction-diffusion equation with state constraints"
17:10 – 17:45 Günter Leugering – FAU Erlangen-Nürnberg: "Time-domain decomposition of optimal control problems for systems of semilinear hyperbolic equations"
17:50 – 18:00 [Closing]

INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

Abstracts

ROMA • ONLINE | JULY 1 - 7, 2021

BOOK OF ABSTRACTS

INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

Abstracts

- Piermarco Cannarsa – University of Roma "Tor Vergata": "Bilinear control for evolution equations"
- Masahiro Yamamoto – The University of Tokyo: "Carleman estimates and inverse problems for transport equation"
- Sylvain Ervedoza – University of Bordeaux: "Observability of waves in an annulus for various boundary conditions"
- Judith Vancostenoble – Paul Sabatier University: "Lipschitz stability for the growth rate coefficients in a nonlinear Fisher-KPP equation"
- Giuseppe Buttazzo – University of Pisa: "Upper and lower bounds for some shape functionals"
- Elisabetta Rocca – University of Pavia: "Optimal control and log-time behavior of diffuse interface models of tumor growth"
- Roberta Bianchini – CNR, Italy: "Some recent results on 2D stably stratified fluids"
- Fabio Camilli – Sapienza University of Roma: "A regularity theory for viscous Hamilton-Jacobi equations with Caputo time-fractional derivative"
- Paola Loreti – Sapienza University of Roma: "Memory Effects and PDEs in Glass Relaxation"
- Cristina Urbani – University of Roma "Tor Vergata": "A constructive algorithm for building mixing coupling potentials. Application to bilinear control"*
- Biagio Ricceri – University of Catania: "Recent results on variational inequalities"
- Cristina Pignotti – University of L'Aquila: "Consensus of opinion formation models with time delay"
- Alessandro Paolucci – University of L'Aquila: "On the control of the Hegselmann-Krause model with leadership and time delay"*
- Martin Lazar – University of Dubrovnik: "Optimal control of parabolic equations – a spectral calculus based approach"
- Umberto Biccari – DeustoCCM | University of Deusto: "Multilevel Selective Harmonic Modulation via Optimal Control"*
- Francesca Bucci – University of Firenze: "On an optimal evolution in linear-quadratic control"
- Cristian Mendico – GSSI L'Aquila, Italy: "Ergodic behavior of solutions to Hamilton-Jacobi equations for sub-Riemannian control systems"
- Sandra Carillo – Sapienza University of Roma: "The integro-differential model of viscoelastic body: thermodynamically admissible kernels"
- Alfio Quarteroni – Mox, Politecnico of Milano: "The Integrated Heart"
- Gianluigi Rozza – SISSA, Trieste: "An Overview of Reduced Order Methods for Optimal Flow Control Problems"
- Karine Beauchard – ENS Rennes: "On expansions for nonlinear systems, error estimates and convergence issues"
- Marco Caponigro – University of Roma "Tor Vergata": "Exact controllability in projections of the bilinear Schrödinger equation"
- Lucio Boccardo – Sapienza University of Roma: "Regularizing effect of the interplay between coefficients in Dirichlet problems with convection or drift terms"
- Emmanuel Trélat – Sorbonne University: "Robustness of various controllability properties under sampling"
- Camille Pouchol – University of Paris: "Unwanted sparsity for linear inverse problems with non-negativity constraints"*
- Martin Gugat – FAU Erlangen-Nürnberg: "Some remarks on the turnpike property"
- Michael Hintermüller – Humboldt Berlin University: "Optimization with learning-informed differential equation constraints and its applications"

INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

Abstracts

- Alessio Porretta – University of Roma "Tor Vergata": "Effects of gradient blow-up in viscous Hamilton-Jacobi equations"
- Maurizio Falcone – Sapienza University of Roma: "Dynamic Programming on a tree for the approximation of finite horizon optimal control problems"
- Lorena Bociu – NC State University: "Multiscale interface coupling between a poroelastic medium and a lumped hydraulic circuit"
- Pierluigi Colli – University of Pavia: "The viscous Cahn-Hilliard system and the sliding mode control problem"
- Cristina Trombetti – University of Naples Federico II: "An optimization problem in thermal insulation"
- Dongnam Ko – The Catholic University of Korea: "Model predictive control with random batch methods for a guiding problem"*
- Yavar Kian – Aix-Marseille University: "Recovery of nonlinear terms for reaction-diffusion equations"
- Emmanuelle Crepeau – University of Grenoble: "Internal null controllability of the Hirota-Satsuma system"
- Genni Fragnelli – University of Bari: "Boundary Controllability for a degenerate wave equation in non divergence form with drift"
- Roberto Guglielmi – University of Waterloo, Canada: "Sensitivity analysis of the value function for semilinear parabolic optimal control problems"*
- Teresa Scarinci – University of L'Aquila, Italy: "Some optimal control problems under uncertainty and convergence analysis for stochastic proximal gradient methods"*
- Harbir Antil – George Mason University: "Fractional Operators: Analysis and Optimal Control with Physical and Data Science Applications"
- Nicola Bellomo – University of Granada | Polytechnic of Torino: "Modeling Virus Pandemics by Multiscale Methods"
- Enrico Valdinoci – The University of Western Australia: "The Lévy flight foraging hypothesis"
- Christophe Zhang – FAU Erlangen-Nürnberg: "Shape control of the heat equation"*
- Marius Tucsnak – University of Bordeaux, France: "Controllability of a class of infinite dimensional systems with age structure"
- Martin Burger – FAU Erlangen-Nürnberg: "Kinetic Models and (Social) Networks"
- Éric Soccorsi – Aix-Marseille University: "Do quantum currents uniquely determine the magnetic potential of an Iwatsuka Hamiltonian?"
- Oleg Y. Imanuvilov – Colorado State University: "Controllability to trajectories of a simplified fluid-structure interaction model"
- Hiroshi Takase – University of Tokyo: "Inverse problems for first-order hyperbolic equations"*
- Domènec Ruiz-Balet – UAM | DeustoCCM: "Control of reaction-diffusion equation with state constraints"
- Günter Leugering – FAU Erlangen-Nürnberg: "Time-domain decomposition of optimal control problems for systems of semilinear hyperbolic equations"

Bilinear control for evolution equations

Piermarco Cannarsa
University of Rome Tor Vergata
cannarsa@axp.mat.uniroma2.it

Bilinear control systems are receiving increasing attention in recent years, as they can be used to study problems for which an additive control action would be unrealistic. For such systems, in infinite dimension, weaker controllability properties can be expected than for systems with additive controls. For instance, exact controllability is out of question due to a well-known negative result by Ball, Marsden, and Slemrod back in the 80's. Nevertheless, one can seek to steer states to special targets either in finite or infinite time. In this talk, I will present recent results where the above problem is addressed for evolution equations of the form $u'(t) = Au(t) + p(t)Bu(t)$, with A and B linear operators in a Hilbert space and $p(t)$ a single-input control. Applications to parabolic equations in one space dimension will also be discussed.

Joint work with: Fatiha Alabau-Boussouira (*Sorbonne Université and Université de Lorraine*), Cristina Urbani (*University of Rome Tor Vergata*)

Carleman estimates and inverse problems for transport equation

Masahiro YAMAMOTO

Graduate School of Mathematical Sciences, The University of Tokyo

E-mail:**myama@ms.u-tokyo.ac.jp**

Abstract

We consider a transport equation of the first order in a bounded smooth domain $\Omega \subset \mathbb{R}^d$:

$$\partial_t u(x,t) + (H(x,t)\cdot \nabla u(x,t)) + p(x,t)u(x,t) = F(x,t), \quad x \in \Omega,\, 0 < t < T, \qquad (1)$$

where $H \in C([0,T];C^1(\overline{\Omega})^d)$.

We consider two topics.

- global Carleman estimates in $\Omega \times (0,T)$
- Applications to inverse problems: estimates global in the domain

Our approach is global in x and see [2] as for a local approach.

In order to establish the stability in L^2-spaces for inverse problems of determining for example a spatial factor in (1) by boundary data, the Carleman estimate is relevant. The method of characteristics is not convenient for L^2-estimates of solutions and moreover the transport equation often appears coupled with parabolic systems in fluid dynamics, so that the characteristics does not work for such studies.

Part I.

A Carleman estimate can be established directly if the rotation angle $H(x,t)$ over $\Omega \times (0,T)$ is

smaller than π (I will formulate in my talk). In the case where the rotational angle is greater than or equal to π, we make a suitable partition of Ω and $0 < t < T$ into smaller subdomains and construct suitable piecewise weight functions to establish a Carleman estimate in $\Omega \times (0,T)$. The partition of Ω requires us to make favorable estimation on interfaces, and we must assume some outgoing-incoming conditions associated with $H(x,t)$ on some interfaces. We illustrate such interface conditions of H and prove a Carleman estimate.

Part II.

By Huang, Imanuvilov and Yamamoto [4] which modifies a classical methodology (Bukhgeim and Klibanov [1]), we demonstrate that a Carleman estimate yields the stability for inverse source problems and inverse coefficient problems and also the observability inequality.

This is a joint work with Professor Piermarco Cannarsa (University of Rome Tor Vergata) and Professor Giuseppe Floridia (Università Mediterranea di Reggio Calabria).

References

[1] A.L. Bukhgeim and M.V. Klibanov, Uniqueness in the large of a class of multidimensional inverse problems, Soviet Math. Dokl. **24** (1981) 244-247.

[2] P. Cannarsa, G. Floridia, F. Gögeleyen, and M. Yamamoto, Inverse coefficient problems for a transport equation by local Carleman estimate, Inverse Problems **35** (2019) 105013 22 pp.

[3] F. Gölgeleyen and M. Yamamoto, Stability for some inverse problems for transport equations, SIAM J. Math. Anal. **48** (2016) 2319–2344.

[4] X. Huang, O.Y. Imanuvilov and M. Yamamoto, Stability for inverse source problems by Carleman estimates, Inverse Problems **36** (2020) 125006, 20 pp.

Observability of waves in an annulus for various boundary conditions.

Sylvain Ervedoza
Institut de Mathématiques de Bordeaux UMR 5251, Université de Bordeaux, Bordeaux INP, CNRS, F-33400 Talence, France.
sylvain.ervedoza@math.u-bordeaux.fr

In this talk, I will present a study of observability properties of the wave equation in an annulus when the condition on the inner sphere is a general dynamic boundary condition. In particular, we will give some sufficient conditions guaranteeing observability of the model at hand. In order to do this, we will develop an approach based on appropriate resolvent estimates, some multiplier techniques and the factorization of the operators. We will also discuss the necessity of these conditions by explicit constructions.

This talk is based on a joint work with Lucie Baudouin, Jérémi Dardé, and Alberto Mercado.

Joint work with: Lucie Baudouin (*LAAS, Univ. de Toulouse*), Jérémi Dardé (*IMT, Univ. de Toulouse*), and Alberto Mercado (*Departamento de Mathemática, Univ. Tecnica Federico Santa Maria*).

Lipschitz stability for the growth rate coefficients in a nonlinear Fisher-KPP equation

Judith Vancostenoble
Université Paul Sabatier Toulouse III
vancoste@math.univ-toulouse.fr

We consider a reaction-diffusion model of biological invasion in which the evolution of the population is governed by several parameters among them the intrinsic growth rate. The knowledge of this growth rate is essential to predict the evolution of the population, but it is a priori unknown for exotic invasive species. We prove uniqueness and unconditional Lipschitz stability for the corresponding inverse problem, taking advantage of the positivity of the solution inside the spatial domain and studying its behaviour near the boundary with maximum principles. Our results complement previous works by Cristofol and Roques.

References

[1] M. Cristofol and L. Roques, Biological invasions : deriving the regions at risk from partial measurements, *Mathematical Biosciences*, **Volume no. 215** (2008), pp. 158-166.

[2] M. Cristofol and L. Roques, Stable estimation of two coefficients in a nonlinear Fisher-KPP equation, *Inverse Problems*, **Volume no. 29** (2013), 18 pp.

[3] P. Martinez and J. Vancostenoble, Lipschitz stability for the growth rate coefficients in a nonlinear Fisher-KPP equation, *Discrete Contin. Dyn. Syst. Ser. S*, **Volume no. 14** (2021), pp. 695–721.

Joint work with: Patrick Martinez (*Université Paul Sabatier Toulouse III*)

Upper and lower bounds for some shape functionals

Giuseppe Buttazzo
University of Pisa
giuseppe.buttazzo@unipi.it

The relations between some quantities related to the Laplace operator are considered. In particular, principal eigenvalue and torsional rigidity are studied in the class of general domains, convex domains, and domains with a small thickness. This allows to obtain a detailed description of the Blasche-Santal diagram of the two quantities. Several open questions are discussed, in particular when the Laplacian is replaced by the p-Laplacian.

References

[1] M. van den Berg, G. Buttazzo: *On capacity and torsional rigidity.* Bull. Lond. Math. Soc., **53** (2021), 347–359.

[2] M. van den Berg, G. Buttazzo, A. Pratelli: *On the relations between principal eigenvalue and torsional rigidity.* Commun. Contemp. Math., (to appear), preprint available at http://cvgmt.sns.it and at http://www.arxiv.org

[3] L. Briani, G. Buttazzo, F. Prinari: *Some inequalities involving perimeter and torsional rigidity.* Appl. Math. Optim., (to appear), preprint available at http://cvgmt.sns.it and at http://www.arxiv.org

[4] L. Briani, G. Buttazzo, F. Prinari: *A shape optimization problem on planar sets with prescribed topology.* Preprint available at http://cvgmt.sns.it and at http://www.arxiv.org

[5] G. Buttazzo, A. Pratelli: *An application of the continuous Steiner symmetrization to Blaschke-Santaló diagrams.* ESAIM Control Optim. Calc. Var., (to appear), preprint available at http://cvgmt.sns.it and at http://www.arxiv.org

Optimal control and log-time behavior of diffuse interface models of tumor growth

Elisabetta Rocca
University of Pavia
elisabetta.rocca@unipv.it

In this talk we report about the joint work with Cecilia Cavaterra and Hao Wu [1]. We investigate the long-time dynamics and optimal control problem of a thermodynamically consistent diffuse interface model that describes the growth of a tumor in presence of a nutrient and surrounded by host tissues. The state system consists of a Cahn–Hilliard type equation for the tumor cell fraction and a reaction–diffusion equation for the nutrient. The possible medication that serves to eliminate tumor cells is in terms of drugs and is introduced into the system through the nutrient. In this setting, the control variable acts as an external source in the nutrient equation. First, we consider the problem of "long-time treatment" under a suitable given mass source and prove the convergence of any global solution to a single equilibrium as $t \to +\infty$ Second, we consider the "finite-time treatment" that corresponds to an optimal control problem. Here we allow the objective cost functional to depend on a free time variable, which represents the unknown treatment time to be optimized.We prove the existence of an optimal control and obtain first order necessary optimality conditions for both the drug concentration and the treatment time. One of the main aim of the control problem is to realize in the best possible way a desired final distribution of the tumor cells, which is expressed by the target function Φ. By establishing the Lyapunov stability of certain equilibria of the state system (without external source), we show that Φ can be taken as a stable configuration, so that the tumor will not grow again once the finite-time treatment is completed.

References

[1] C. Cavaterra, E. Rocca, H. Wu, Long-time Dynamics and Optimal Control of a Diffuse Interface Model for Tumor Growth, *Applied Mathematics Optimization*, **Volume no. 83** (2021), pp. 739–787.

Joint work with: Cecilia Cavaterra (*University of Milan*), Hao Wu (*Fudan University*).

Some recent results on 2D stably stratified fluids

Roberta Bianchini
Consiglio Nazionale delle Ricerche CNR - IAC
`roberta.bianchini@cnr.it`

This talk will concern the analysis of the stably stratified fluids system, where the velocity satisfies the incompressible Euler equations, coupled with a scalar term, called *buoyancy*. It is obtained by a linearization of the equations of incompressible non-homogeneous fluids around a *background density profile* $\bar{\rho}(z)$ that increases with depth. Adding the *Boussinesq approximation*, according to which density variation is neglected except when it directly causes buoyancy forces, leads to the Boussinesq system. In the first part, I will describe the properties of the smooth solutions to the 2D Boussinesq equations with a damping term in the velocity equation. This is based on a joint work with Roberto Natalini (CNR-IAC, Rome), [3]. The second part will be devoted to the presentation of the stability properties of the 2D Boussinesq equations around the Couette flow $(y, 0)$. The last part is based on a joint work with Michele Coti Zelati (Imperial College, UK) and Michele Dolce (Imperial College, UK) [2] and a joint work with Jacob Bedrossian (University of Maryland, USA), Michele Coti Zelati and Michele Dolce [1].

References

[1] J. Bedrossian, R. Bianchini, M. Coti Zelati, M. Dolce, Nonlinear inviscid damping and shear-buoyancy instability in the two-dimensional Boussinesq equations; preprint arXiv:2103.13713v1 (2021).

[2] R. Bianchini, M. Coti Zelati, M. Dolce, Linear inviscid damping for shear flows near Couette in the 2D stably stratified regime, *Indiana University Math. Journal*, in press; preprint arXiv 2005.09058 (2020).

[3] R. Bianchini, R. Natalini, Asymptotic behavior of 2D stably stratified fluids with a damping term in the velocity equation, *ESAIM: COCV*, in press; preprint arXiv:2009.01578 (2020).

A regularity theory for viscous HamiltonJacobi equations with Caputo time-fractional derivative

Fabio Camilli
Sapienza Università di Roma
fabio.camilli@uniroma1.it

We present a regularity theory for classical and weak solutions to viscous Hamilton-Jacobi equations with Caputo time-fractional derivative. The approach relies on a combination of gradient bounds for the time-fractional Hamilton-Jacobi equation obtained via Evans's nonlinear adjoint method and sharp estimates in Sobolev and Hölder spaces for the corresponding linear problem.

Joint work with: Alessandro Goffi (*Università di Padova*)

Memory Effects and PDEs in Glass Relaxation

Paola Loreti
Sapienza Universit di Roma
Dipartimento di Scienze di Base e Applicate per l'Ingegneria
paola.loreti@uniroma1.it

The interest devoted to glass materials is mainly concerned with high-tech applications regarding the best possible performances for computer displays.

For some glass relaxation models the stretched exponential function, obtained by inserting a fractional power into the exponential, has been proposed as stress relaxation modulus.

The stretched exponential function is routinely employed to describe relaxation in glasses and in glass-forming liquids.

Motivated by the the goodness of the approximation of the stretched exponential function with Prony series [2], we discuss the class of integro-differential equations

$$u_{tt} = \triangle u - \sum_{i=1}^{N} b_i \int_0^t e^{-r_i(t-s)} \triangle u(s) ds$$

and also an Ingham type approach to the related observability problem ([1] and [3]).

References

[1] P. Loreti, D. Sforza, Viscoelastic aspects of glass relaxation models, *Phys. A*, **Volume no.526** (2019), 10 pp.

[2] J. C. Mauro, Y. Z. Mauro, On the Prony series representation of stretched exponential relaxation, *Phys. A*, **Volume no.506** (2018), pp. 7587.

[3] P. Loreti, D. Sforza, Inverse Ingham type inequalities for the Burgers model, in preparation.

Joint work with: Daniela Sforza (*Sapienza Universit di Roma, Dipartimento di Scienze di Base e Applicate per l'Ingegneria*)

A constructive algorithm for building mixing coupling potentials. Application to bilinear control

Cristina Urbani
University of Rome Tor Vergata
urbani@mat.uniroma2.it

Despite the importance of control systems governed by a bilinear control for the description of phenomena that could not be modeled by additive controls, stabilization and controllability problems for such kind of systems have not been so widely studied in the literature as it happens for boundary and locally distributed controls. The main reasons of this fact might be found in the intrinsic nonlinear nature of such problems and furthermore, for controls that are scalar functions of time, in an ineluctable obstruction for exact controllability under very general assumptions presented in the celebrated work of Ball, Marsden and Slemrod [3].

By violating one of the hypotheses of [3], Beauchard and collaborators have succeeded in proving exact controllability for hyperbolic systems in a topology stronger than the natural one for such problems [4,5]. Whereas, for evolution equations of parabolic type, results of rapid stabilization and exact controllability to eigensolutions have been demonstrated by Alabau-Boussouira, Cannarsa and Urbani [1,2]. A common step that can be found in all the aforementioned works is the proof of the exact controllability of the system obtained by linearization of the nonlinear problem along the reference trajectory. It turns out that a necessary condition on the rank of the potential must be satisfied. However, even if the genericity of such assumption has been proved for instance in [5], it is in practice difficult to exhibit examples of suitable potentials.

The aim of this talk is to present a constructive algorithm that provides a infinite class of functions that fulfill the required property.

References

[1] F. Alabau-Boussouira, P. Cannarsa, C. Urbani, Exact controllability to eigensolutions for evolution equations of parabolic type via bilinear control, *preprint available on arXiv:2105.05732*, (2021)

[2] F. Alabau-Boussouira, P. Cannarsa, C. Urbani, Superexponential stabilizability of evolution equations of parabolic type via bilinear control, *Journal of Evolution Equations*, **Volume no. 20** (2020)

[3] J.M. Ball, J.E. Marsden, M. Slemrod, Controllability for distributed bilinear systems, *SIAM Journal on Control and Optimization*, **Volume no. 20, n. 4** (1982), pp. 575-597

[4] K. Beauchard, Local controllability and non-controllability for a 1D wave equation with bilinear control, *Journal of Differential Equations*, **Volume no. 250, n. 4** (2011), pp. 2064-2098

[5] K. Beauchard, C. Laurent, Local controllability of 1D linear and nonlinear Schrödinger equations with bilinear control, *J. Math. Pures Appl.*, **Volume no. 94, n. 5** (2010), pp. 520-554

Joint work with: Fatiha Alabau-Boussouira (*Université de Lorraine & Laboratoire JLL, Sorbonne Université*)

Recent results on variational inequalities

Biagio Ricceri
Department of Mathematics and Informatics, University of Catania
ricceri@dmi.unict.it

In this lecture, I will deal with two recent results on variational inequalities. One of them reads as follows:

Let $(H, \langle \cdot, \cdot \rangle)$ *be a real Hilbert space and let* $\Phi : B_\rho \to H$ *be a* $C^{1,1}$ *function, with* $\Phi(0) \neq 0$*. Then, for each* $r > 0$ *small enough, there exists a unique point* $x^* \in S_r$*, such that*

$$\max\{\langle \Phi(x^*), x^* - x\rangle, \langle \Phi(x), x^* - x\rangle\} < 0$$

for all $x \in B_r \setminus \{x^*\}$*, where*

$$B_r = \{x \in H : \|x\| \leq r\}$$

and

$$S_r = \{x \in H : \|x\| = r\} \ .$$

References

[1] B. Ricceri, An improvement of a saddle point theorem and some of its applications, (2021), to appear on *J. Nonlinear Convex Anal.*

[2] J. Saint Raymond, A theorem on variational inequalities for affine mappings, *Minimax Theory Appl.*, **4** (2019), 281-304.

Consensus of opinion formation models with time delay

Cristina Pignotti
Università di L'Aquila, Italy
cristina.pignotti@univaq.it

We will analyze a class of opinion formation models with time delay. Under appropriate assumptions, we will show the exponential asymptotic consensus when the time delay satisfies a suitable smallness assumption. Our result combines a Lyapunov functional approach with careful estimates on the trajectories. We then study the mean-field limit from the many-individual equation to the continuity-type partial differential equation as the number N of individuals goes to infinity. For the limiting equation, we prove global-in-time existence and uniqueness of measure-valued solutions. We also use the fact that constants appearing in the consensus estimates for the particle system are independent of N to extend the exponential consensus result to the continuum model.

References

[1] Y.-P. Choi, A. Paolucci and C. Pignotti, Consensus of the Hegselmann-Krause opinion formation model with time delay, *Math. Meth. Appl. Sci.*, **44** (2021), pp. 4560-4579.

Joint work with: Young-Pil Choi (*Yonsei University, Seoul, Republic of Korea*), Alessandro Paolucci (*Università di L'Aquila*).

On the control of the Hegselmann-Krause model with leadership and time delay.

Alessandro Paolucci
Università degli Studi dell'Aquila
alessandro.paolucci2@graduate.univaq.it

In this talk we present a Hegselmann-Krause opinion formation model with leadership in presence of time delay effects. In particular, we consider a model with pointwise time variable time delay and a model with a distributed delay. In both cases we show that, when the delays and the parameters involved in the dynamics satisfy suitable smallness conditions, then the leader can control the system, leading the group to any prefixed state, which gives us the local controllability result for the Hegselmann-Krause model. Some numerical tests will illustrate our analytical results.

References

[1] A. Paolucci and C. Pignotti, On the control of the Hegselmann-Krause model with leadership and time delay, *Preprint 2021*, arXiv:2105.14248.

Joint work with: Cristina Pignotti (*Università degli Studi dell'Aquila*).

Optimal control of parabolic equations - a spectral calculus based approach.

Martin Lazar
University of Dubrovnik
mlazar@unidu.hr

Abstract: We consider an optimal control problem for a general linear parabolic equation governed by a self-adjoint operator on an abstract Hilbert space. The task consists in identifying a control (entering the system through the initial condition) that minimises a given cost functional, while steering the final state close to the given target. This can be considered as an inverse problem (of initial source identification) for parabolic equations from the optimal control viewpoint.

In order to efficiently deal with this problem, we propose a novel approach based on the spectral calculus for self adjoint operators and geometrical representation of the problem. We obtain closed form expression for the control solution as a function of the operator governing the dynamics of the system. Its numerical computation is performed by exploring efficient Krylov subspace techniques, by which one constructs a rational approximation of the aforementioned function of the operator.

The efficiency of the proposed algorithm method is confirmed through numerical examples, which will be also presented.

Joint work with: Luka Grubišić, Ivica Nakić (*University of Zagreb*), and Martin Tautenhahn (*University of Leipzig*).

Multilevel Selective Harmonic Modulation via Optimal Control

Umberto Biccari
Fundación Deusto & Universidad de Deusto
Bilbao, Spain
umberto.biccari@deusto.es

We consider the Selective Harmonic Modulation (SHM) problem, consisting in the design of a staircase control signal with some prescribed frequency components. We propose a novel methodology to address SHM as an optimal control problem in which the admissible controls are piece-wise constant functions, taking values only in a given finite set. In order to fulfill this constraint, we introduce a cost functional with piece-wise affine penalization for the control, which, by means of Pontryagin's maximum principle, makes the optimal control have the desired staircase form. Moreover, the addition of the penalization term for the control provides uniqueness and continuity of the solution with respect to the target frequencies. We also provide several numerical examples in which the SHM problem is solved by means of our approach.

References

[1] D. J. Oroya Villalta, C. Esteve-Yagüe and U. B., Multilevel Selective Harmonic Modulation via Optimal Control, preprint (2021), arXiv:2103.10266

Joint work with: Deyviss Jesús Oroya Villalta (*Universidad de Deusto*) and Carlos Esteve-Yagüe (*Universidad Autónoma de Madrid*).

On an optimal evolution in linear-quadratic control

Francesca Bucci
Università degli Studi di Firenze, Italy
francesca.bucci@unifi.it

The optimal boundary control problem with quadratic functionals for linear partial differential equations (PDE) over an infinite time horizon generally yields an optimal state which fulfils the semigroup property. In the case of parabolic (and parabolic-like) PDE in bounded domains, the regularity properties of the optimal state semigroup as well as the description of its infinitesimal generator's domain – which have a role in connection with the Riccati equations that correspond to the optimization problem – are shaped by the analyticity of the operator that describes the free dynamics.

In this talk we will report recent progress concerning the said generator, within a framework which is consistent with the boundary control of coupled systems of hyperbolic-parabolic PDE instead. The analytic tools that are needed and prove effective are the powers of positive operators and interpolation, besides the distinguishing setting that characterizes the controlled dynamics.

Joint work with: Paolo Acquistapace (*Università di Pisa*)

Ergodic behavior of solutions to Hamilton-Jacobi equations for sub-Riemannian control systems

Cristian Mendico
GSSI-Gran Sasso Science Institute & CEREMADE Paris Dauphine
cristian.mendico@gssi.it

In the recent years, increasing attention has been devoted to control systems of the form

$$\dot{\gamma}(t) = \sum_{i=1}^{m} u_i f_i(\gamma(t)) \qquad (1)$$

where f_i are $m \in \{1, \ldots, d\}$ vector fields on $\mathbb{R}^d$ and u_i are measurable controls on $\mathbb{R}^m$. Indeed, they naturally appears when one needs to model systems in which agents can move only along fixed directions.

In this talk we will discuss some recent results on the long time behavior of solutions to the Hamilton-Jacobi equations associated with systems of type (1). In particular, we will show that the time-average value function converges, as the time horizon goes to infinity, to the ergodic constant. Then, we address the existence of solutions to the ergodic Hamilton-Jacobi equations by analyzing the Abel means. Finally, we define and study the properties of the Lax-Oleinik semigroup proving also the existence of a solution to the ergodic equation which is a fixed-point of such semigroup.

References

[1] Cannarsa, P. and Mendico, C., Asymptotic analysis for Hamilton-Jacobi equations associated with sub-Riemannian control systems, *Arxiv:*2012.09099.

Joint work with: Piermarco Cannarsa, (*University of Rome Tor Vergata*).

The integro-differential model of viscoelastic body: thermodynamically admissible kernels

Sandra Carillo
Dipartimento Scienze di Base e Applicate per l'Ingegneria
Sapienza Università di Roma, 16, Via A. Scarpa, 00161 Rome, Italy
and
Gr. Roma1, IV - Mathematical Methods in NonLinear Physics
National Institute for Nuclear Physics (I.N.F.N.), Rome, Italy
sandra.carillo@uniroma1.it

Materials with memory are modelled via integro-differential equations exhibiting an integral term which takes into account the past *history* of the material. A body is termed *viscoelastic* when its mechanical response is determined not only by the present status but also by its deformation history. The problem under investigation, when u denotes the displacement and f an external term which takes into account also the history of the material, reads

$$u_{tt} = G(0)u_{xx} + \int_0^t \dot{G}(t-\tau)u_{xx}(\tau)d\tau + f \tag{1}$$

wherein the kernel of the integral term represents the relaxation function G which characterises the viscoelastic material.

The *classical* regularity requirements [1] the relaxation function is assumed to satisfy are reconsidered. The aim is to relax them to model wider classes of materials. Various cases [2], [3], [4], thermodynamically admissible, are considered.

The results presented are mainly concerned about a joint research program with M. Chipot, V. Valente and G. Vergara Caffarelli. Further investigations are currently in progress [5].

References

[1] C.M. Dafermos, An abstract Volterra equation with applications to linear viscoelasticity, *J. Diff. Equations*, **7** (1970), pp. 554-569

[2] S. Carillo, V. Valente, G. Vergara Caffarelli A linear viscoelasticity problem with a singular memory kernel: an existence and uniqueness result, *Differential and Integral Equations*, **26** (2013), pp. 1115-1125

[3] S. Carillo, A 3-dimensional singular kernel problem in viscoelasticity: an existence result, *Atti della Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche, Matematiche e Naturali*, **97** (S1), A3 (2019), 13 pp.

[4] S. Carillo, M. Chipot, V. Valente, G. Vergara Caffarelli, On weak regularity requirements of the relaxation modulus in viscoelasticity, *Communications in Applied and Industrial Mathematics*, **10** (1) (2019), 78-87

[5] S. Carillo, M. Chipot, V. Valente, G. Vergara Caffarelli, in progress (2021).

The Integrated Heart

Alfio Quarteroni
Politecnico di Milano, Italy and
Ecole Polytechnique Fédérale de Lausanne, Switzerland
alfio.quarteroni@polimi.it

Mathematical models based on first principles allow the description of the blood motion in the human circulatory system, as well as the interaction between electrical, mechanical and fluid-dynamical processes occurring in the heart. This is a classical environment where multi-physics processes have to be addressed. Appropriate numerical strategies can be devised to allow for the analysis of both heart function and dysfunction, and the simulation, control and optimization of therapy and surgery. This presentation will address some of these issues and a few representative applications of clinical interest.

An Overview of Reduced Order Methods for Optimal Flow Control Problems

Gianluigi Rozza
mathLab, Mathematics Area, SISSA, Via Bonomea 265, I-34136 Trieste, Italy
gianluigi.rozza@sissa.it

This talk focuses on strategies and results dealing with reduced order methods (ROMs) for parametrized optimal flow control problems (OFCP(μ)s) governed by partial differential equations (PDE(μ)s), which can bridge the gap between collected data and model, described by physical and/or geometrical parameters. If on one side OFCP(μ)s are a very versatile tool, on the other, their applicability is still limited due to the costs of their simulations. To overcome this issue, we rely on ROMs. First of all, we will describe a space-time POD-Galerkin algorithm [1] suited to several governing equations. A first coastal management application described by Shallow Waters is shown [2]. Then, we will introduce space-time Greedy based on a new error estimator for parabolic OFCP(μ)s tested with a boundary control over a Graetz flow [3]. In the last part of the talk, we will highlight the potential of such methodologies within advanced applications, such as uncertainty quantification [4] and OFCP(μ)s to steer bifurcating phenomena in nonlinear systems [5].

References

[1] M. Strazzullo, F. Ballarin, and G. Rozza, "POD–Galerkin Model Order Reduction for Parametrized Time Dependent Linear Quadratic Optimal Control Problems in Saddle Point Formulation", *Journal of Scientific Computing*, **83(3)**, pp. 55, 2020.

[2] M. Strazzullo, F. Ballarin, G. Rozza, "POD-Galerkin Model Order Reduction for Parametrized Nonlinear Time Dependent Optimal Flow Control: an Application to Shallow Water Equations", *submitted*.

[3] M. Strazzullo, F. Ballarin, and G. Rozza, "A Certified Reduced Basis Method for Linear Parametrized Parabolic Optimal Control Problems in Space-Time Formulation", *submitted*.

[4] G. Carere, M. Strazzullo, F. Ballarin, G. Rozza, R. Stevenson. "Weighted POD-reduction for parametrized PDE-constrained Optimal Control Problems with random inputs and its applications to environmental sciences", *submitted*.

[5] F. Pichi, M. Strazzullo, F. Ballarin, and G. Rozza "Driving bifurcating parametrized nonlinear PDEs by optimal control strategies: application to Navier-Stokes equations and model reduction", *submitted*.

Joint work with: Maria Strazzullo (*mathLab, Mathematics Area, SISSA, Via Bonomea 265, I-34136 Trieste, Italy*), Francesco Ballarin (*Department of Mathematics and Physics, Catholic University of the Sacred Heart*).

On expansions for nonlinear systems, error estimates and convergence issues

Karine Beauchard
ENS Rennes, France
karine.beauchard@ens-rennes.fr

Explicit formulas expressing the solution to non-autonomous differential equations are of great importance in many application domains such as control theory or numerical operator splitting. In particular, intrinsic formulas allowing to decouple time-dependent features from geometry-dependent features of the solution have been extensively studied. First, we give a didactic review of classical expansions for formal linear differential equations, including the celebrated Magnus expansion (associated with coordinates of the first kind) and Sussmann's infinite product expansion (associated with coordinates of the second kind). Inspired by quantum mechanics, we introduce a new mixed expansion, designed to isolate the role of a time-invariant drift from the role of a time-varying perturbation. Second, in the context of nonlinear ordinary differential equations driven by regular vector fields, we give rigorous proofs of error estimates between the exact solution and finite approximations of the formal expansions. In particular, we derive new estimates focusing on the role of time-varying perturbations. For scalar-input systems, we derive new estimates involving only a weak Sobolev norm of the input. Third, we investigate the local convergence of these expansions. We recall known positive results for nilpotent dynamics and for linear dynamics. Nevertheless, we also exhibit arbitrarily small analytic vector fields for which the convergence of the Magnus expansion fails, even in very weak senses. We state an open problem concerning the convergence of Sussmann's infinite product expansion. Eventually, we derive approximate direct intrinsic representations for the state and discuss their link with the choice of an appropriate change of coordinates.

Joint work with: Jeremy Le Borgne and Frederic Marbach

Exact controllability in projections of the bilinear Schrödinger equation

Marco Caponigro
Dipartimento di Matematica, Università degli Studi di Roma "Tor Vergata", Viale della Ricerca Scientifica 1, 00133 Rome, Italy
caponigro@mat.uniroma2.it

We e present a general method for the approximate controllability of the bilinear Schrödinger equation based on Lie-algebraic control techniques applied to suitable finite dimensional approximation of the Galerkin-type. Under some regularity assumptions on the Hamiltonians and generic conditions on the controllability of the finite dimensional Galerkin approximations we show exact controllability in projection on an arbitrary given number of eigenstates.

References

[1] N. Boussaïd, M. Caponigro, T. Chambrion, *Regular propagators of bilinear quantum systems*, Journal of Functional Analysis Volume 278, Issue 6, 1 April 2020, 108412.

[2] M. Caponigro, M. Sigalotti, *Exact controllability in projections of the bilinear Schrödinger equation*, SIAM J. Control Optim., 56(4), 2018, pp 29012920.

Regularizing effect of the interplay between coefficients in Dirichlet problems with convection or drift terms

Lucio Boccardo
Sapienza Università di Roma - Istituto Lombardo
boccardo@mat.uniroma1.it

There are very important results by **Enrique Zuazua** on the subject of the convection-diffusion equation

$$u_t - \operatorname{div}(a(x)\nabla u) = -d \cdot \nabla(|u|^{q-1}u), \quad \text{in } (0,+\infty) \times R^N.$$

The second part of the talk follows, side by side, the paper [2], dedicated to "Enrike-60, and it deals with a linear (i.e. $q = 1$) stationary counterpart of the above equation if d is not constant. We prove regularizing results on the solutions, under assumptions of **interplay** between the datum and the coefficient of the zero order term or between the modulus of the drift and the coefficient of the zero order term.

The first part of the talk is devoted to the introduction to the two roots needed:

- the Dirichlet problems, "formally" in duality, (studied in [3,...,8])

  $$(CP) \quad u \in W_0^{1,2}(\Omega): \ -\operatorname{div}(M(x)\nabla u) + a(x)\, u = -\operatorname{div}(u\, E(x)) + f(x),$$

  $$(DP) \quad \psi \in W_0^{1,2}(\Omega): \ -\operatorname{div}(M(x)\nabla \psi) + a(x)\, \psi = E(x) \cdot \nabla\psi + g(x),$$

  where Ω is a bounded open set in R^N, $M(x)$ ia bounded elliptic matrix, f, g are functions belonging to L^m, $m \geq 1$, $|E| \in L^N$, $0 < \alpha_0 \leq a(x) \in L^1$;

- the regularizing effect, in the Dirichlet problem

  $$u: \ -\operatorname{div}(M(x)\nabla u) + a(x)\, u = f(x),$$

  of the **interplay** assumption $|f(x)| \leq Q\, a(x) \in L^1$, which implies that u belongs to $W_0^{1,2}(\Omega) \cap L^\infty(\Omega)$ (see [1]).

Then, for parabolic problems, whose simplest model is

$$\begin{cases} u_t(x,t) - \Delta u = -\operatorname{div}(u(x,t)\, E(x,t)) & \text{in } Q_T \equiv \Omega \times (0,T), \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega\,, \end{cases}$$

is proved the estimate (see [9])

$$\|u(t)\|_{L^q} \leq C_E \frac{\|u_0\|_{L^1}}{t^{\frac{N}{2}(1-\frac{1}{q})}} \qquad \text{for every } t \text{ in } (0,T).$$

References

[1] D. Arcoya, L. Boccardo: J. Funct. Anal. (2015).

[2] L. Boccardo: ESAIM Control Optim. Calc. Var. (2021).

[3] L. Boccardo: Boll. Unione Mat. Ital. (2009).

[4] L. Boccardo: J. Differential Equations (2015).

[5] L. Boccardo: Differential Integral Equations (2019).

[6] L. Boccardo: ESAIM Control Optim. Calc. Var. (2019).

[7] L. Boccardo, S. Buccheri, G.R. Cirmi: NoDEA (2020).

[8] L. Boccardo, S. Buccheri: Rev. Mat. Complut. (2021).

[9] L. Boccardo, L. Orsina, M.M. Porzio: J. Evolution Equations (2021).

Robustness of various controllability properties under sampling

Emmanuel Trélat
Sorbonne Université
emmanuel.trelat@sorbonne-universite.fr

I will report on a series of works done with L. Bourdin, with whom we have investigated the following general question: given a control system, do we have have robustness under control sampling of various controllability or stabilization properties?

This question has been much investigated in the past, in particular by K. Grasse, E. Sontag, H. Sussmann. I will give the state-of-the-art and provide new results.

References

[1] L. Bourdin, E. Trélat, *Robustness under control sampling of reachability in fixed time for nonlinear control systems*, Preprint Hal (2020), 16 pages.

[2] L. Bourdin, E. Trélat, *Unified Riccati theory for optimal permanent and sampled-data control problems in finite and infinite time horizons*, SIAM J. Control Optim. **59** (2021), no. 2, 489–508.

[3] L. Bourdin, E. Trélat, *Linear-quadratic optimal sampled-data control problems: convergence result and Riccati theory*, Automatica J. IFAC **79** (2017), 273–281.

[4] L. Bourdin, E. Trélat, *Optimal sampled-data control, and generalizations on time scales*, Math. Control Relat. Fields **6** (2016), no. 1, 53–94.

Joint work with: Loïc Bourdin (*Univ. Limoges*).

Unwanted sparsity for linear inverse problems with non-negativity constraints

Camille Pouchol
MAP5, Université de Paris, Paris, France.
camille.pouchol@u-paris.fr

The purpose of this talk is the analysis of linear inverse problems $A\mu = y$ with non-negative constraints on the unknown μ. The applicative context is medical imaging, and the purpose is to reconstruct a (reasonably smooth) image.

Depending on the noise statistics, the problem is usually solved by

$$\min_{\mu \geq 0} D(y, A\mu),$$

where D is some appropriate divergence, or a regularisation thereof. Without sufficient regularisation, these problems are known to lead to unwanted spikes in the reconstructed image.

Working in the space of non-negative Radon measures, I will explain why this is typical of large noise: any optimal measure will have a singular part as soon as $y \notin \{A\mu, \mu \geq 0\}$. This requires both suitable hypotheses on the operator A and the divergence D. Dual arguments allow to partially locate where spikes will arise [1].

If time permits, I will discuss the case of the Kullback-Leibler divergence in more detail, in particular when the operator A models Positron Emission Tomography [2].

References

[1] C.P., Olivier Verdier, Linear inverse problems with nonnegativity constraints through the β-divergences: sparsity of optimisers, *Arxiv preprint arXiv:2006.15845* (2020),

[2] C.P., Olivier Verdier, The ML-EM algorithm in continuum: sparse measure solutions, *Inverse Problems*, **36** (2020).

Joint work with: Olivier Verdier (*Department of Computing, Electrical Engineering and Mathematical Sciences, Western Norway University of Applied Sciences, Bergen, Norway*)

Some remarks on the turnpike property

Martin Gugat
Department of Data Sciences, Friedrich-Alexander University Erlangen-Nuremberg
martin.gugat@fau.de

The turnpike phenomenon is a structural property or the solutions of optimal control problems that occurs if the optimal control problems have a certain structure which is often called dissipativity. A typical example are dynamic optimal control problems with tracking terms in the objective functional, where a certain desired state appears. Turnpike results give statements about the distance between the optimal trajectories and a certain static state (the *turnpike*). Often the turnpike can by found as the solution of certain static optimal control problems.

In this talk we comment about the finite-time turnpike phenomenon, which is also called exact turnpike, where the turnpike state is reached exactly after finite time. This situation occurs if non-smooth tracking terms appear in the objective functional Moreover we discuss the phenomenon in the presence of probabilistic constraints.

References

[1] M. Gugat, E. Zuazua, and M. Schuster, The finite-time turnpike phenomenon for optimal control problems: Stabilization by non-smooth tracking terms, in *Stabilization of Distributed Parameter Systems: Design Methods and Applications*, A. Z. Grigory Sklyar, ed., vol. 2 of SEMA SIMAISpringer Series ICIAM 2019, Springer Nature Switzerland, 2021, pp. 1741.

Optimization with learning-informed differential equation constraints and its applications

Michael Hintermüller
Weierstrass Institute for Applied Analysis and Stochastics, Berlin
hintermueller@wias-berlin.de

Inspired by applications in optimal control of semilinear elliptic partial differential equations and physics-integrated imaging, differential equation constrained optimization problems with constituents that are only accessible through data-driven techniques are studied. A particular focus is on the analysis and on numerical methods for problems with machine-learned components. For a rather general model problem, an error analysis is provided, and particular properties resulting from artificial neural network based approximations are addressed. Moreover, for each of the two inspiring applications analytical details are presented and numerical results are provided. In this context we are interested in two scenarios: (i) a nonlinear constituent of a PDE is learned from data and combined with *a initio* model components. The resulting hybrid PDE model then serves as a constraint in a minimization problem. (ii) The entire parameter/control-to-state map is learned from data and used for reducing the underlying optimization problem. In both cases, the learning phase is considered offline which allows to focus on the optimal control resp. inverse problem.

References

[1] G. Dong, M. Hintermüller, K. Papafitsoros, Optimization with learning-informed differential equation constraints and its applications, *Preprint Weierstrass Institute Berlin*, **No. 2754** (2020), submitted to COCV.

Joint work with: Guozhi Dong (*Weierstrass Institute Berlin*), Kostas Papafitsoros (*Weierstrass Institute Berlin*)

Effects of gradient blow-up in viscous Hamilton-Jacobi equations

Alessio Porretta
University of Rome Tor Vergata
porretta@mat.uniroma2.it

This talk is concerned with the diffusive Hamilton-Jacobi equation

$$\begin{cases} u_t - \Delta u = |\nabla u|^p & (x,t) \in \Omega \times (0,T) \\ u = 0 & (x,t) \in \partial\Omega \times (0,T) \\ u(x,0) = u_0 & x \in \Omega \end{cases}$$

in the *superquadratic growth* regime, say $p > 2$, for a bounded domain Ω and a given continuous initial condition u_0.

It has been understood, in the recent years, that this equation shows very peculiar phenomena when $p > 2$, due to the competition between second order and first order terms. In particular, it is known that classical solutions may not exist globally in time, due to gradient blow-up, which can only occur at the boundary ([2]). One of the effects of the boundary gradient blow-up is the possible loss of the prescribed boundary condition ([1]). However, in long time the solution will become smooth again, recovering the boundary condition and, eventually, converging to zero ([3]). When and how shall we expect the gradient blow-up to happen and, in particular, to cause loss of the boundary condition? And how shall we expect the solution to recover from this relaxed behavior and return to smoothness? Jointly with Philippe Souplet, we give some answers to those questions, which involve the blow-up (and smoothness recovery) rate, the evolution of the boundary behavior and, after all, a precise description in one-dimensional cases ([4], [5]).

References

[1] G. Barles and F. Da Lio, On the generalized Dirichlet problem for viscous Hamilton–Jacobi equations, *J.* Math. Pures Appl. **83** (2004), 53-75.

[2] Ph. Souplet and Q.S. Zhang, Global solutions of inhomogeneous Hamilton–Jacobi equations, *J. Anal. Math.* **99** (2006), pp. 355-396.

[3] A. Porretta and E. Zuazua, Null controllability of viscous Hamilton–Jacobi equations, *Ann. IHP, A-N* **29** (2012), pp 301-333.

[4] A. Porretta and P. Souplet, Analysis of the loss of boundary condition for a viscous Hamilton-Jacobi equation, *A*nn. IHP, A-N **34** (2017), 1913-1923.

[5] A. Porretta and P. Souplet,, Blow-up and regularization rates, loss and recovery of boundary conditions for the superquadratic viscous Hamilton-Jacobi equation, *J.* Math. Pures Appl. **133** (2020), 66-117.

Joint work with: Philippe Souplet(*Université Paris 13, France*)

Dynamic Programming on a tree for the approximation of finite horizon optimal control problems

M. Falcone
Dipartimento di Matematica
Università di Roma "La Sapienza"
e-mail: `falcone@mat.uniroma1.it`

The classical Dynamic Programming (DP) approach to optimal control problems is based on the characterization of the value function as the unique viscosity solution of a Hamilton-Jacobi-Bellman (HJB) equation [2]. The DP scheme for the numerical approximation of viscosity solutions of those equations is typically based on a time discretization which is projected on a fixed space triangulation of the numerical domain [3]. The time discretization can be done by a one-step scheme for the dynamics and the projection on the grid typically uses a polynomial interpolation. This approach, which allows to get information on optimal controls in feedback form, has been shown to be very powerful for low dimensional optimal control problems although general convergence results have been proved in $\mathbb{R}^d$. Several methods have been proposed to mitigate the curse of dimensionality of DP schemes, e.g. static and dynamic domain decomposition, fast-marching and fast-sweeping methods, discrete representation formulas (when available), see [3] and the references therein.
We will discuss a new approach for finite horizon optimal control problems [1, 4] where we compute the value function on a tree structure generated by the time discrete dynamics avoiding the construction of a space grid/triangulation to solve the HJB equation. This allows to drop the cost of the space interpolation, moreover the tree will guarantee a perfect matching with the discrete dynamics. We prove first order convergence to the value function for a first order discretization of the dynamics. We will also discuss extensions to high-order schemes and to problems with state constraints showing also some numerical tests.

Work in collaboration with A. Alla (PUC, Rio de Janeiro) and L. Saluzzi (Sapienza, Roma).

References

[1] A. Alla, M. Falcone and L. Saluzzi. *An efficient DP algorithm on a tree-structure for finite horizon optimal control problems*, SIAM Journal on Scientific Computing, (41) 4, 2019, A2384-A2406

[2] M. Bardi, I. Capuzzo-Dolcetta, *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*, Birkhäuser, Basel, 1997.

[3] M. Falcone, R. Ferretti, *Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi Equations*, Society for Industrial and Applied Mathematics, Philadelphia, 2013.

[4] L. Saluzzi, A. Alla and M. Falcone. *Error estimates for a tree structure algorithm for dynamic programming equations*, submitted, 2018 `https://arxiv.org/abs/1812.11194`

Multiscale interface coupling between a poroelastic medium and a lumped hydraulic circuit

Lorena Bociu
NC State University
lvbociu@ncsu.edu

We consider a multiscale problem modeling the flow of a fluid through a deformable porous medium, described by poroelasticity, connected with a lumped hydraulic circuit, described by a system of ordinary differential equations. The motivation comes from biological models where the perfusion of a specific tissue or organ (local phenomenon) is studied in correlation to the global features of the surrounding blood circulation. Mathematically, this PDE/ODE coupled problem includes nonlocal interface conditions enforcing the continuity of mass and the balance of stresses across models at different scales. We address questions related to the solution methods of the PDE/ODE coupled problem, focusing on a detailed comparison between functional iterations and an energy-based operator splitting method and how they handle the nonlocal interface conditions. We provide sufficient conditions for the convergence of functional iterations and prove that the energy-based operator splitting method is unconditionally stable with respect to the size of the time discretization step [1].

References

[1] L. Bociu, G. Guidoboni, D. Prada, and R. Sacco, Numerical simulation and analysis of multiscale interface coupling between a poroelastic medium and a lumped hydraulic circuit: comparison between functional iteration and operator splitting methods, *Computer Methods in Applied Mechanics and Engineering*, (2021), submitted.

Joint work with: Giovanna Guidoboni (*University of Missouri, USA*), Daniele Prada (*Consiglio Nazionale delle Ricerche, Pavia, Italy*), and Riccardo Sacco (*Politecnico di Milano, Italy*)

The viscous Cahn-Hilliard system and the sliding mode control problem

Pierluigi Colli
Dipartimento di Matematica 'F. Casorati'
Università di Pavia
Via Ferrata 5, 27100 Pavia, Italy
pierluigi.colli@unipv.it

The talk deals with a viscous Cahn-Hilliard system in which an additional leading term in the expression for the chemical potential is present. This term consists of a subdifferential operator S acting, in the Lebesgue space $L^2(\Omega)$, on the difference between the phase variable and a prescribed state depending on space and time. Existence and continuous dependence results are discussed for either homogeneous Neumann or Dirichlet boundary conditions for the chemical potential. Next, letting S be a multiple of the sign operator and assuming Dirichlet boundary conditions, we also prove the sliding mode property in a suitable setting: by this the phase variable is forced to join the evolution of the given state in some time lower than the given final time T.

The presentation is based on the results that have been obtained in a collaboration with Gianni Gilardi and Gabriela Marinoschi.

An optimization problem in thermal insulation

Cristina Trombetti
Università degli studi di Napoli Federico II
cristina@unina.it

We study thermal insulating of a bounded body. Under a prescribed heat source, we consider a model of heat transfer between the body and the environment determined by convection; this corresponds, before insulation, to Robin boundary conditions. We study the maximization of heat content (which measures the goodness of the insulation) among all the possible distributions of insulating material with fixed mass, and prove an optimal upper bound in terms of geometric quantities. Eventually we prove a conjecture which states that the ball surrounded by a uniform distribution of insulating material maximizes the heat content.

Joint work with: Francesco della Pietra, (*Università degli studi di Napoli Federico II*), Carlo Nitsch, (*Università degli studi di Napoli Federico II*), Riccardo Scala(*Università degli studi di Siena*).

Model predictive control with random batch methods for a guiding problem

Dongnam Ko
The Catholic University of Korea
dongnamko@catholic.ac.kr

We model, simulate and control the guiding problem for a herd of evaders under the action of repulsive drivers. The problem is formulated in an optimal control framework, where the drivers (controls) aim to guide the evaders (states) to a desired region of the Euclidean space. The numerical simulation of such models quickly becomes unfeasible for a large number of interacting agents, as the number of interactions grows $O(N^2)$ for N agents. For reducing the computational cost to $O(N)$, we use the Random Batch Method (RBM), which provides a computationally feasible approximation of the dynamics. For this approximated dynamics, the corresponding optimal control can be computed efficiently using a classical gradient descent. The resulting control is not optimal for the original system, but for a reduced RBM model. We therefore adopt a Model Predictive Control (MPC) strategy to handle the error in the dynamics. Through numerical experiments we show that the combination of RBM and MPC leads to a significant reduction of the computational cost, preserving the capacity of controlling the overall dynamics.

References

[1] D. K. and E. Zuazua, Model predictive control with random batch methods for a guiding problem, preprint (2021), to appear on *Math. Models Methods Appl. Sci.*, arXiv:2004.14834

Joint work with: Enrique Zuazua (*FAU Erlangen-Nürnberg, Deusto Foundation-Bilbao, and UAM-Madrid*)

Recovery of nonlinear terms for reaction-diffusion equations

Yavar Kian
Aix-Marseille University
yavar.kian@univ-amu.fr

In this talk, we consider the inverse problem of determining a general semilinear term appearing in a nonlinear parabolic equation. Our goal is to determine the nonlinear term from lateral boundary measurements of solutions of the equation with initial condition fixed at zero. For this purpose, we derive a new criterion allowing to prove such a result for some general class of semilinear terms. This talk is based on some joint work with Gunther Uhlmann.

Internal null controllability of the Hirota-Satsuma system

Emmanuelle Crépeau
LJK-Université Grenoble Alpes
`emmanuelle.crepeau@univ-grenoble-alpes.fr`

The generalized Hirota-Satsuma system consists of three coupled nonlinear Korteweg-de Vries (KdV) equations. By using two distributed controls, we prove that the local null controllability property holds when the system is posed on a bounded interval. First, the system is linearized around the origin obtaining two decoupled subsystems of third order dispersive equations. This linear system is controlled with two inputs, which is optimal. This result is obtained with a duality approach and some appropriate Carleman estimates. Then, by means of an inverse function theorem, the local null controllability of the nonlinear system is proven.

References

[1] Araruna, F. D., Cerpa, E., Mercado, A. and Santos, M. C., Internal null controllability of a linear Schrödinger–KdV system on a bounded interval, *Journal of Differential Equations*, **260(1)**, (2016), pp.653-687.

[2] Capistrano–Filho, R. A., Pazoto, A. F. and Rosier, L., Internal controllability of the Korteweg–de Vries equation on a bounded domain, *ESAIM: Control, Optimisation and Calculus of Variations*, **21(4)**, (2015),pp. 1076-1107.

[3] Carreño, N., Cerpa, E. and Crépeau, E., Internal null controllability of the generalized Hirota-Satsuma system, *ESAIM: Control, Optimisation and Calculus of Variations*, **26**, (2020), pp.75:1-22.

Joint work with: Nicolás Carreño(*Departamento de Matemática, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile. E-mail:* `nicolas.carrenog@usm.cl`) and Eduardo Cerpa (*Departamento de Matemática, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile. E-mail:* `eduardo.cerpa@usm.cl`)

Boundary Controllability for a degenerate wave equation in non divergence form with drift

Genni Fragnelli
University of Bari Aldo Moro
genni.fragnelli@uniba.it

We consider the problem

$$\begin{cases} u_{tt} - a(x)u_{xx} - b(x)u_x = 0, & (t,x) \in Q, \\ u(t,0) = 0, \quad u(t,1) = f(t), & t \in [0,+\infty), \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where $Q = (0,+\infty) \times (0,1)$, $f \in L^2_{\text{loc}}[0,+\infty)$, $a, b \in C^0[0,1]$, $a > 0$ on $(0,1]$ and $a(0) = 0$. Concerning b, it can possibly degenerate at 0 Here, the function f acts as a boundary control and it is used to drive the solution to 0 at a given time. Clearly the presence of the drift term leads us to use different spaces with respect to the ones in [1] and it gives rise to some new difficulties. However, thanks to some suitable assumptions on the drift term, one can prove some estimates that are crucial to prove an observaibility similar to

$$E_\varphi(0) \leq C_T \int_0^T |\varphi_x(t,1)|^2 dt,$$

where E_φ denotes the energy associated to the solution of the adjoint problem.

References

[1] F. Alabau-Boussouira and P. Cannarsa and G. Leugering, Control and stabilization of degenerate wave equations, *SIAM J. Control Optim.*, **Volume no. 55** (2017), pp. 2052-–2087

Joint work with: Idriss Boutaayamou (*Université Ibn Zohr, Morocco*), Dimitri Mugnai (*Tuscia University, Italy*)

Sensitivity analysis of the value function for semilinear parabolic optimal control problems

Roberto Guglielmi
University of Waterloo
rguglielmi@uwaterloo.ca

We derive sufficient conditions for expressing the first and second order sensitivities of the value function associated with a class of optimal controls problems constrained by parabolic semilinear equations. In particular, we relate the first order sensitivity with the adjoint equation and the second order sensitivity with a suitably defined Riccati equation.

References

[1] Guglielmi R., Kunisch K., Sensitivity analysis for the value function of a semilinear parabolic equation and its relation to Riccati equations, *Optimization*, **67** (2018), 1461-1485.

Joint work with: Karl Kunisch (*University of Graz and RICAM, Linz, Austria*).

Some optimal control problems under uncertainty and convergence analysis for stochastic proximal gradient methods

Teresa Scarinci
Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila
teresa.scarinci@univaq.it

Firstly, we will introduce some PDE-constrained optimization problems with random data and robust formulation in the same fashion as in [KS,L,GS] and references therein. For these models, we will analyse the convergence of a stochastic proximal gradient method. Namely, we show that the iterate produced by the methods will convergence (in a suitable sense) to the stationary points of the optimization problems. After that, an optimal control problem subject to uncertainty and ambiguity will be introduced. We present the differences and difficulties with respect to the robust approach and discuss some research questions.

References

[KS] Kouri, D.P., Surowiec, T.M., Risk-averse PDE-constrained optimization using the conditional value-at-risk, (2016) SIAM Journal on Optimization, 26 (1), pp. 365-396.

[L] G. Lord, C. Powell, and T. Shardlow, An Introduction to Computational Stochastic PDEs, Cambridge University Press, 2014.

[GS] Geiersbach, C., Scarinci, T., Stochastic proximal gradient methods for nonconvex problems in Hilbert spaces, Computational Optimization and Applications, 2021, 78(3), pp. 705–740

Fractional Operators: Analysis and Optimal Control with Physical and Data Science Applications

Harbir Antil
Center for Mathematics and Artificial Intelligence and
Department of Mathematical Sciences
George Mason University
Fairfax, Virginia, 22030 USA
hantil@gmu.edu

Fractional calculus and its application to anomalous diffusion has recently received a tremendous amount of attention. In complex/heterogeneous material mediums, the long-range correlations or hereditary material properties are presumed to be the cause of such anomalous behavior. Owing to the revival of fractional calculus, these effects are now conveniently modeled by fractional-order differential operators and the governing equations are reformulated accordingly.

In the first part of the talk, we plan to introduce both linear and nonlinear, fractional-order differential equations. As applications, we will develop new physical models for geophysical electromagnetism, imaging science and a new notion of optimal control and inverse problems will be discussed. We also plan to introduce a novel variable order fractional Laplacian operator with multiple applications.

In the second part of the talk, we will focus on novel Deep Neural Networks (DNNs) based on fractional operators. We plan to discuss the approximation properties and apply them to image denoising and tomographic reconstruction problems. We will establish that these DNNs are also excellent surrogates to PDEs and inverse problems with multiple advantages over the traditional methods. If time permits, we will conclude the talk by showing some of our initial results on chemically reacting flows using DNNs which clearly shows the effectiveness of the proposed approach.

Modeling Virus Pandemics by Multiscale Methods

Nicola, Bellomo
University of Granada, Spain and Politecnico of Torino, Italy
nicola.bellomo@polito.it

This lecture is devoted to the multidisciplinary modelling of a pandemic initiated by an aggressive virus, specifically the so-called *SARS–CoV–2 Severe Acute Respiratory Syndrome, corona virus n.2.* The study is developed within a multiscale framework accounting for the interaction of different spatial scales, from the small scale of the virus itself and cells, to the large scale of individuals and further up to the collective behaviour of populations. An interdisciplinary vision is developed thanks to the contributions of epidemiologists, immunologists and economists as well as those of mathematical modellers.

We present further developments of the model [1] with the aim firstly, to show how relaxations of the confinement rules can generate sequential waves and, subsequently, the dynamics of mutations into new variants can be modeled. Simulations are developed to provide an description of these dynamics to support the decision making of crisis managers [2]. The overall content offers a perspective towards optimization and control problems somehow related to vaccination programs.

References

[1] N. Bellomo, R. Bingham, M. Chaplain, G. Dosi, G. Forni, D. Knopoff, J. Lowengrub, R. Twarock, M. E. Virgillito, A multi-scale model of virus pandemic: Heterogeneous interactive entities in a globally connected world, *Mathematical Models and Methods in Applied Sciences*, **30** (2020), pp. 1591–1651.

[2] N. Bellomo, D. Burini, N. Outada, Multiscale models of Covid-19 with mutations and variants, *Submitted*, (2021).

Joint work with: Diletta Burini, (*University of Perugia, Italy*) and Nisrine Outada, (*IRD-Sorbonne, France and Cadi Ayyad University, Morocco*)

The Lévy flight foraging hypothesis

Enrico Valdinoci
University of Western Australia
enrico.valdinoci@uwa.edu.au

We consider a forager diffusing via a fractional heat equation and we introduce several efficiency functionals whose optimality is discussed in relation to the Lévy exponent of the evolution equation. Several biological scenarios, such as a target close to the forager, a sparse environment, a target located away from the forager and two targets are specifically taken into account.

The optimal strategies of each of these configurations are here analyzed explicitly also with the aid of some special functions of classical flavor and the results are confronted with the existing paradigms of the Lévy foraging hypothesis.

Interestingly, one discovers bifurcation phenomena in which a sudden switch occurs between an optimal (but somehow unreliable) Lévy foraging pattern of inverse square law type and a less ideal (but somehow more secure) classical Brownian motion strategy. Additionally, optimal foraging strategies can be detected in the vicinity of the Brownian one even in cases in which the Brownian one is pessimizing an efficiency functional.

References

[1] Serena Dipierro, Giovanni Giacomin and Enrico Valdinoci, *Efficiency functionals for the Lévy flight foraging hypothesis*, preprint.

Joint work with: Serena Dipierro (*University of Western Australia*), Giovanni Giacomin (*University of Western Australia*).

Shape control of the heat equation

Christophe Zhang
Chair in Applied Analysis (Alexander von Humboldt professorship), Friedrich Alexander Universität Erlangen-Nürnberg
christophe.zhang@polytechnique.org

We study the following control problem for the linear heat equation on a domain $\Omega \subset \mathbb{R}^d$ (introduced in [1]):

$$\begin{cases} y_t - \Delta y = \chi_{\omega(t)}, \\ y = 0 \text{ on } \partial\Omega \end{cases} \tag{1}$$

where the control is the shape of the subdomain $\omega(t)$, subject to the following constraint:

$$|\omega(t)| \leq L|\Omega|, \quad \forall t > 0. \tag{2}$$

Control problems of this form appear in many different applications, including shape optimization, wavemakers [2] (with fluid equations), and tumor growth models [3].

Combining a relaxation approach with a well-chosen optimal control problem, we prove the following approximate controllability result: nonnegative states are approximately reachable from 0.

The proof relies on the so-called bathtub principle, and some positivity properties of solutions to the linear heat equation. The control is obtained by applying abstract Fenchel-Rockafellar duality arguments.

References

[1] G.Lance, E.Trélat and E.Zuazua, Shape turnpike for linear parabolic PDE models, *Systems & Control Letters*, **142** (2020).

[2] H.Nersisyan, D.Dutykh and E.Zuazua, Generation of 2d water waves by-moving bottom disturbances, *IMA Journal of Applied Mathematics 80*, **4** (2015),1235–1253.

[3] B.Perthame, F.Quirós, and J. L.Vázquez, The Hele–Shaw asymptotics formechanical models of tumor growth, *Archive for Rational Mechanics and Analysis*, **1** (2014), 93–127.

Joint work with: Camille Pouchol (*MAP5, Université de Paris*), Emmanuel Trélat (*LJLL, Sorbonne Université*), Enrique Zuazua (*Chair in Applied Analysis (AvH professorship), FAU Erlangen-Nürnberg*).

CONTROLLABILITY OF A CLASS OF INFINITE DIMENSIONAL SYSTEMS WITH AGE STRUCTURE

DEBAYAN MAITY, MARIUS TUCSNAK, AND ENRIQUE ZUAZUA

Given a linear dynamical system, we investigate the linear infinite dimensional system obtained by grafting an age structure. Such systems appear essentiallyt in population dynamics with age structure when phenomena like spatial diffusion or transport are also taken into consideration. We first show that the new system preserves some of the wellposedness properties of the initial one. Our main result asserts that if the initial system is null controllable in a time small enough than the structured system is also null controllable in a time depending on the various involved parameters.

More precisely, let $A : \mathcal{D}(A) \to X$ be the generator of the C^0 semigroup $\mathbb{S}$ on the Hilbert space$X$ and let U be another Hilbert space. Both X and U will be identified with their duals. Let B be a (possibly unbounded) linear operator from U to X, which is supposed admissible control operator for $\mathbb{S}$. In the examples we have in mind, the above spaces and operators describe the dynamics of a system without age structure. In particular, X is the state space and U is the control space. The corresponding age structured system is obtained by first extending these spaces to

$$\mathcal{X} = L^2(0, a_\dagger; X), \tag{0.1}$$

$$\mathcal{U} = L^2(0, a_\dagger; U), \tag{0.2}$$

where $a_\dagger > 0$ denotes the maximal age individuals can attain. Let $p(t) \in \mathcal{X}$ be the distribution density of the individuals with respect to age $a \geqslant 0$ and at some time $t \geqslant 0$. Then the abstract version of the Lotka-McKendrick system to be considered in this paper writes:

$$\begin{cases} \dfrac{\partial p}{\partial t} + \dfrac{\partial p}{\partial a} - Ap + \mu(a)p = \mathbb{1}_{(a_1,a_2)}Bu, & t \geqslant 0, a \in (0, a_\dagger), \\ p(t, 0) = \displaystyle\int_0^{a_\dagger} \beta(s)p(t, s) \, \mathrm{d}s, & t \geqslant 0, \\ p(0, a) = p_0, \end{cases} \tag{0.3}$$

where $\mathbb{1}$ is the characteristic function of the interval (a_1, a_2) with $0 \leqslant a_1 < a_2 \leqslant a_\dagger$ and p_0 is the initial population density. In the above system, the positive function $\mu : [0, a_\dagger] \to \mathbb{R}_+$ denotes the natural mortality rate of individuals of age a. We denote by $\beta : [0, a_\dagger] \to \mathbb{R}_+$ the positive function describing the fertility rate at age a. We assume that the fertility rate β and the mortality rate μ satisfy the conditions

(H1) $\beta \in L^\infty(0, a_\dagger)$, $\beta \geqslant 0$ for almost every $a \in (0, a_\dagger)$.
(H2) $\mu \in L^\infty[0, a^*]$ for every $a^* \in (0, a_\dagger)$, $\mu \geqslant 0$ for almost every $a \in (0, a_\dagger)$.
(H3) $\displaystyle\int_0^{a_\dagger} \mu(a) \, \mathrm{d}a = +\infty$.

For more details about the modelling of such system and the biological significance of the hypotheses, we refer to Webb [9].

Before we state our main result, let us introduce the notion of null controllability of the pair (A, B).

Date: June 28, 2021.

Definition 0.1. *We say that a pair (A, B) is null-controllable in time τ, if for every $z_0 \in X$ there exists a control $u \in L^2(0, \tau, U)$ such that, the solution of the system*

$$\dot{z}(t) = Az(t) + Bu(t) \quad t \in [0, \tau], \qquad z(0) = z_0,$$

satisfies $z(\tau) = 0$.

The main result of this paper is:

Theorem 0.2. *Assume that β and μ satisfy the conditions (H1)-(H3) above. Moreover, suppose that the fertility rate β is such that*

$$\beta(a) = 0 \text{ for all } a \in (0, a_b), \tag{0.4}$$

for some $a_b \in (0, a_\dagger)$ and that $a_1 < a_b$. Let us assume that the pair (A, B) is null controllable in time $\tau > \tau_0$, with

$$0 \leqslant \tau_0 \leqslant \overline{\tau}, \quad \overline{\tau} = \min\{a_2 - a_1, a_b - a_1\}. \tag{0.5}$$

Then for every $\tau > a_1 + a_\dagger - a_2 + 2\tau_0$ and for every $p_0 \in \mathcal{X}$ there exists a control $v \in L^2(0, \tau; \mathcal{U})$ such that the solution p of (0.3) satisfies

$$p(\tau, a) = 0 \text{ for all } a \in (0, a_\dagger). \tag{0.6}$$

This result can be seen as a generalization of those obtained in [2, 3, 1, 7, 8] in the case when A is an elliptic operator with Neumann or Dirichlet homogeneous boundary conditions or in Ainseba et al. [4], Boutaayamou et al.[5] or Fragnelli [6] when A is a degenerate elliptic operator. Our approach applies, besides the above mentioned examples, to operators A such that the systems without age structure describes fractional diffusion, transport phenomena or even Schrödinger type dynamics, with internal or boundary control.

The proof of the above theorem relies on final time observability of the adjoint system. This consists of combining characteristics method with final state observability of the pair (A^*, B^*), with no reference to the methodology employed to prove this observability result for the system without age structure. This idea was already used in [8] where A was second order elliptic differential operator and B was interior control operator.

References

[1] B. Ainseba, *Exact and approximate controllability of the age and space population dynamics structured model*, J. Math. Anal. Appl., 275 (2002), pp. 562–574.

[2] B. Ainseba and S. Aniţa, *Local exact controllability of the age-dependent population dynamics with diffusion*, Abstr. Appl. Anal., 6 (2001), pp. 357–368.

[3] B. Ainseba and S. Aniţa, *Internal exact controllability of the linear population dynamics with diffusion*, Electron. J. Differential Equations, (2004), pp. No. 112, 11.

[4] B. Ainseba, Y. Echarroudi, L. Maniar, et al., *Null controllability of a population dynamics with degenerate diffusion*, Differential and integral equations, 26 (2013), pp. 1397–1410.

[5] I. Boutaayamou and Y. Echarroudi, *Null controllability of a population dynamics with interior degeneracy*, arXiv preprint arXiv:1704.00936, (2017).

[6] G. Fragnelli, *Carleman estimates and null controllability for a degenerate population model*, Journal de Mathématiques Pures et Appliquées, 115 (2018), pp. 74–126.

[7] N. Hegoburu and M. Tucsnak, *Null controllability of the Lotka-McKendrick system with spatial diffusion*. working paper or preprint, Nov. 2017.

[8] D. Maity, M. Tucsnak, and E. Zuazua, *Controllability and Positivity Constraints in Population Dynamics with Age Structuring and Diffusion*. working paper or preprint, May 2018.

[9] G. F. Webb, *Theory of nonlinear age-dependent population dynamics*, vol. 89 of Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, Inc., New York, 1985.

Kinetic Models and (Social) Networks

Martin Burger
FAU Erlangen-Nürnberg
martin.burger@fau.de

In this talk we discuss the derivation of kinetic and subsequent macroscopic equations for processes related to processes on networks, such as opinion formation on social networks. We consider in particular the case when networks are co-evolving during other processes and discuss suitable descriptions as well as issues to derive simple closure relations. Moreover, we discuss aspects of pattern formation such as consensus or the formation of echo chambers.

References

[1] M Burger, Network structured kinetic models of social interactions, *Vietnam J. Math.* **49**(2021), to appear.

[2] M Burger, Kinetic Equations for Processes on Co-evolving Networks (2021), arXiv preprint arXiv:2104.13815

Do quantum currents uniquely determine the magnetic potential of an Iwatsuka Hamiltonian?

Éric Soccorsi
Aix-Marseille Université
eric.soccorsi@univ-amu.fr

An Iwatsuka Hamiltonian is a Schrödinger operator $H_b = (i\nabla - A(x)) \cdot (i\nabla - A(x))$ acting in $L^2(\mathbb{R}^2)$, with magnetic potential $A(x) = \begin{pmatrix} 0 \\ a(x) \end{pmatrix}$ where

$$a(x) = \int_0^x b(\xi) d\xi, \ x \in \mathbb{R}$$

and $b \in L^\infty(\mathbb{R})$ fulfills $\lim_{x\to\pm\infty} b(x) = b_\pm$, $0 < b_- < b_+ < +\infty$.

The spectral and transport properties of H_b have been studied extensively over the last decades, see e.g. [1], and it is well known that Iwatsuka Hamiltonians induce quantum currents which are generated by the non-constancy of the magnetic field b, see [2].

This talk is concerned with the uniqueness issue in the inverse coefficient problem of determining the magnetic field b from knowledge of the quantum currents induced by H_b.

References

[1] P. D. Hislop and É. Soccorsi, Edge states induced by Iwatsuka Hamiltonians with positive magnetic fields, *J. Math. Anal. Appl.*, **422 no. 1** (2015), pp. 594-624

[2] N. Dombrowski, P. D. Hislop and É. Soccorsi, Edge currents and eigenvalue estimates for magnetic barrier Schrödinger operators, *Asympt. Anal.*, **89 no. 3-4** (2014), pp. 331-363

Joint work with: Mourad Choulli (*Université de Lorraine*) and Nour-El-Houda Kerraoui (*Aix-Marseille Université*).

Controllability to trajectories of a simplified fluid-structure interaction model

Oleg, Yu. Imanuvilov
Colorado State University
colostate-oleg@mail.ru

We prove the exact controllability result to trajectories of a simplified model of motion of a rigid body in a fluid flow. Unlike known results such a trajectory does not need to be a stationary solution.

Inverse problems for first-order hyperbolic equations

Hiroshi Takase
Graduate School of Mathematical Sciences, The University of Tokyo
htakase@ms.u-tokyo.ac.jp

Let $d \in \mathbb{N}$ and $\Omega \subset \mathbb{R}^d$ be a bounded domain with Lipschitz boundary $\partial\Omega$. Set $Q := \Omega \times (0,T)$ for $T > 0$. We consider the first-order partial differential operator P such that

$$Pu := A^0(x,t)\partial_t u + A(x,t) \cdot \nabla u,$$

where $A^0 \in C^1(\overline{Q})$ is a positive function and $A = (A^1, \ldots, A^d) \in C^2(\overline{Q}; \mathbb{R}^d)$ is a vector-valued function. Define an outgoing boundary $\Sigma_+ := \{(x,t) \in \partial\Omega \times (0,T) \mid A(x,t) \cdot \nu(x) > 0\}$ and incoming boundary $\Sigma_- := \big(\partial\Omega \times (0,T)\big) \setminus \Sigma_+$, where ν denotes the outer unit normal to $\partial\Omega$. We consider the following two kinds of inverse problems.

Inverse source problem

Let u be a function satisfying

$$\begin{cases} Pu + p(x,t)u = R(x,t)f(x) & \text{in } Q, \\ u = 0 & \text{on } \Sigma_-, \\ u(\cdot, 0) = 0 & \text{on } \Omega, \end{cases}$$

where $p \in W^{1,\infty}(0,T; L^\infty(\Omega))$, $R \in H^1(0,T; L^\infty(\Omega))$, and $f \in L^2(\Omega)$. We will see global Lipschitz stability to determine f in Ω from observation data on Σ_+.

Inverse coefficient problem

For $m = 1, \ldots, d+1$, let u_m be a function satisfying

$$\begin{cases} Pu_m + p(x,t)u_m = 0 & \text{in } Q, \\ u_m = g_m & \text{on } \Sigma_-, \\ u_m(\cdot, 0) = \alpha_m & \text{on } \Omega, \end{cases}$$

where $p \in W^{1,\infty}(0,T; L^\infty(\Omega))$, $g_m \in L^2(\Sigma_-)$, and $\alpha_m \in W^{1,\infty}(\Omega)$. We will see global Lipschitz stability to determine $\{A^\mu\}_{\mu=0}^d$ from finitely many observation data on Σ_+.

References

[1] G. Floridia and H. Takase. Inverse problems for first-order hyperbolic equations with time-dependent coefficients. *arXiv* 2009.12039.

Joint work with Prof. Giuseppe Floridia (*Mediterranea University of Reggio Calabria*).

Control of reaction-diffusion equation with state constraints

Domènec, Ruiz-Balet
Universidad Autónoma de Madrid
domenec.ruizi@uam.es

The classical controllability theory for parabolic equations does not take into account the fact that specific reaction-diffusion equations make physically sense only if the state is positive or, in case that the solution has to model a proportion, taking values in $[0,1]$. We will discuss controllability properties of equations of the type

$$\begin{cases} \partial_t u - \mu\Delta u = f(u) & (x,t) \in \Omega \times (0,T), \\ u(x,t) = a(x,t) & (x,t) \in \partial\Omega \times (0,T), \\ u(0,\cdot) = u_0 \in L^\infty(\Omega;[0,1]), \end{cases}$$

in a way that the controlled trajectory satisfies $0 \leq u(x,t) \leq 1$.

The talk will concern the techniques developed in [1,2,3,4] in order to obtain control strategies satisfying the modeling constraints and related phenomenology. Furthermore. some open perspectives will be exposed.

References

[1] D. Pighin and E. Zuazua, Controllability under positivity constraints of semilinear heat equations., *Mathematical Control & Related Fields*, **8.3&4** (2018), pp. 935

[2] C. Pouchol, E. Trélat and E. Zuazua, Phase portrait control for 1D monostable and bistable reaction–diffusion equations., *Nonlinearity*, **32.3** (2019), pp. 884

[3] D. Ruiz-Balet and E. Zuazua, Control under constraints for multi-dimensional reaction-diffusion monostable and bistable equations., *Journal de Mathématiques Pures et Appliquées*, **143** (2020), pp. 345-275

[4] I. Mazari and D. Ruiz-Balet and E. Zuazua, Constrained control of gene-flow models, *Preprint*,

---

Joint work with: Idriss Mazari (*TU Wien*), Enrique Zuazua (*FAU-Erlangen-Nürnberg, Universidad Autónoma de Madrid, Fundación Deusto*)

Time-domain decomposition of optimal control problems for systems of semilinear hyperbolic equations

Günter Leugering
FAU-Erlangen-Nürnberg
guenter.leugering@fau.de

We extend the time-domain decomposition method described by Lagnese and Leugering [1,2] to semilinear optimal control problems for hyperbolic systems with spatio-temporal varying coefficients. We provide the design of the iterative method applied to the global first-order optimality system, prove its convergence, and derive an a posteriori error estimate. The analysis is done entirely on the continuous level. A distinguishing feature of the method is that the decomposed optimality system can be interpreted as an optimality system of a local *virtual* optimal control problem. Thus, the iterative time-domain decomposition of the optimality system can be interpreted as an iterative parallel scheme for virtual optimal control problems on the subintervals. A typical example and further comments are given to show the range of potential applications. Moreover, we provide some numerical experiments to give a first interpretation of the role of the parameters involved in the iterative process.

References

[1] Lagnese, John E and Leugering, Günter, Time-domain decomposition of optimal control problems for the wave equation, Systems & Control Letters (48) 3-4,229-242

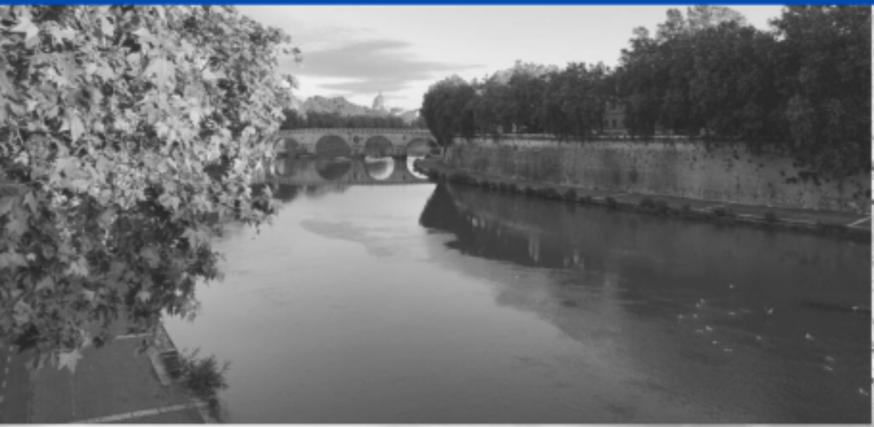
[2] Lagnese, John E. and Leugering, Günter, Domain decomposition methods in optimal control of partial differential equations, International Series of Numerical Mathematics 148, Birkhäuser Verlag, Basel, 2004, xiv+443,

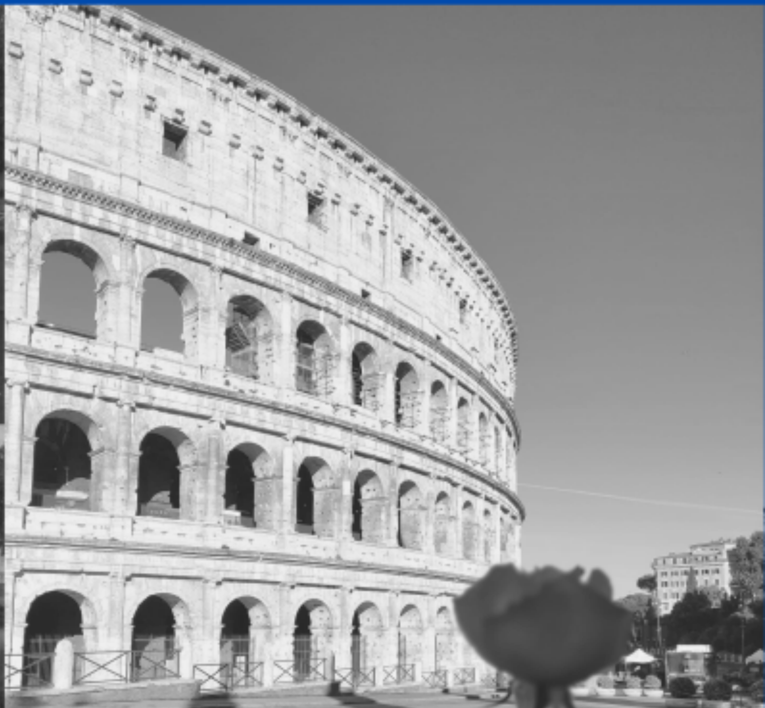
INdAM 2021

ANALYSIS AND NUMERICS OF DESIGN, CONTROL AND INVERSE PROBLEM

This Book of Abstracts has been made by

Darlis Bracho Tudares darlis.bracho@math.fau.de





ROMA • ONLINE
JULY 1 - 7, 2021