

Optimal control and log-time behavior of diffuse interface models of tumor growth

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In this talk we report about the joint work with Cecilia Cavaterra and Hao Wu [1]. We investigate the long-time dynamics and optimal control problem of a thermodynamically consistent diffuse interface model that describes the growth of a tumor in presence of a nutrient and surrounded by host tissues. The state system consists of a Cahn–Hilliard type equation for the tumor cell fraction and a reaction–diffusion equation for the nutrient. The possible medication that serves to eliminate tumor cells is in terms of drugs and is introduced into the system through the nutrient. In this setting, the control variable acts as an external source in the nutrient equation. First, we consider the problem of “long-time treatment” under a suitable given mass source and prove the convergence of any global solution to a single equilibrium as $t \rightarrow +\infty$. Second, we consider the “finite-time treatment” that corresponds to an optimal control problem. Here we allow the objective cost functional to depend on a free time variable, which represents the unknown treatment time to be optimized. We prove the existence of an optimal control and obtain first order necessary optimality conditions for both the drug concentration and the treatment time. One of the main aim of the control problem is to realize in the best possible way a desired final distribution of the tumor cells, which is expressed by the target function Φ . By establishing the Lyapunov stability of certain equilibria of the state system (without external source), we show that Φ can be taken as a stable configuration, so that the tumor will not grow again once the finite-time treatment is completed.

References

- [1] C. Cavaterra, E. Rocca, H. Wu, Long-time Dynamics and Optimal Control of a Diffuse Interface Model for Tumor Growth, *Applied Mathematics Optimization*, **Volume no. 83** (2021), pp. 739–787.

Joint work with: Cecilia Cavaterra (*University of Milan*), Hao Wu (*Fudan University*).